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PSYCHOACOUSTICS AND PASSIVE SONAR
DETECTION

J. M. Stallard, et al

Naval Ordnance Laboratory
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NOLTR 74-27	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PSYCHOACOUSTICS AND PASSIVE SONAR DETECTION		5. TYPE OF REPORT & PERIOD COVERED Final
7. AUTHOR(s) J.M. STALLARD and C.B. LESLIE		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS NAVAL ORDNANCE LABORATORY SILVER SPRING, MARYLAND 20910		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT TASK AREA & WORK UNIT NUMBERS NOL-0910/PMS-4021
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE 9 September 1974
		13. NUMBER OF PAGES 60 67
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) passive sonar acoustics psychoacoustics detectability		
<p style="text-align: center;">Reproduced by NATIONAL TECHNICAL INFORMATION SERVICE U.S. Department of Commerce Springfield, VA. 22151</p>		
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PSYCHOACOUSTICS AND PASSIVE SONAR DETECTION

Prepared by:

J.M. Stallard and C.B. Leslie

SUMMARY

The initial portion of the report is a review of the statistical theory of signal detection, followed by the application of signal detection theory to psychoacoustics. The differences between the relatively simple laboratory tests reported in the literature and the complex problems of passive sonar operating in the real world are explored. The concepts of Equal Detectability Curves and Figure Of Merit are covered, relative to theoretical prediction of alertment and masking performance. It is pointed out that several relevant factors have been treated individually in the literature but not taken into account in the available Detection Threshold Curves for application to passive sonar. The combined effects of five such factors are used to yield a new Detection Threshold between 5 and 6 db higher than earlier works. It is concluded that the problem of sequential observations needs more experimental study. A method for handling the effects of multiple lines in a signature is presented in detail. A new system for calculating the effects of broadband noise of any bandwidth on detectability is given. Finally, suggestions are given for a number of areas where further work might be helpful in eliminating or reducing present uncertainties.

Preface

This report had its origins in a study of the theory of psychoacoustics applied to passive sonar. The results of the initial study prompted a further effort to advance this relatively new field by documenting areas of disagreement with current practice and to present new proposals for handling certain additional factors which have previously been ignored. The authors are indebted to the pioneers in this area, especially B.G. Watters and J.A. Moore. Others who have contributed to the advancement of the theory are J.E. Barger, F.A. Andrews, and C.D. Hovater.

This work has been supported by the Naval Sea Systems Command under Task Number NOL-0910/PMS-4021.

ROBERT WILLIAMSON II

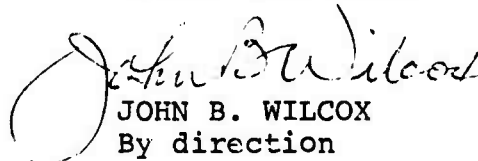

JOHN B. WILCOX
By direction

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CHAPTER 1

INTRODUCTION

The theoretical prediction of the detection range of an underwater acoustic signal received by a human operator through a passive sonar has been a topic of considerable interest to the Navy. The detection range can be related to the maximum transmission loss a signal can suffer and still be detectable to the human ear. This value is called the Figure Of Merit, or FOM, of the system. The passive sonar equation relates the phenomena which must be considered in the calculation of the FOM:

$$\text{FOM} = \text{TL} = \text{SL} - \text{NL} + \text{DI} - \text{DT}. \quad (1-1)$$

In Equation (1-1), TL is the transmission loss, SL is the signal source level, NL is the noise level at the receiving hydrophone, DI is the directivity index of the receiving sonar, and DT is the detection threshold, or the signal to noise ratio that is just detectable. All the quantities are expressed in decibels and the notation is after that of Urick (1967). The sonar equation parameters vary considerably with frequency and the detection threshold will be seen to depend as well on the character of the source.

In this paper we are concerned with the quantification of the detection threshold of the human ear. This quantity is, in general, somewhat higher than that of an ideal device designed to perform a specific task. Despite this shortcoming, the wide range of tasks the human ear can perform, and the fact that the hearing mechanism includes the brain, make the human operator an integral part of many sonars. The detection threshold of the human ear is thus a worthy topic for discussion.

Urick (1967)* cites a simple model for the hearing mechanism which is essentially an ideal detector followed by a post detection averager whose integration time is generally not matched to the signal duration. This simple physical model provides a fairly credible replica of the detection performance of the ear, underestimating the detection threshold by some 4 db, when compared to

*The of references at the end of the report is alphabetical.

data gathered under closely controlled laboratory conditions. It would appear, however, that a truly quantitative model must account for the variation in response of the human hearing mechanism to acoustic signals of different types over a range of frequencies. It must also take into account uncertainties inherent in the real-world passive sonar problem, such as unknown frequency and signal onset time. Recently, Watters and Moore (1970) presented a nomograph which allows the ready determination of either the broadband or tonal signal energy which is just detectable at a given receiver operating level when it is embedded in a background of noise. The basis for their nomograph is a pair of measurements from the literature which grew out of statistical decision theory and the theory of signal detectability, as applied to psychoacoustics. Barger (1971) later used this nomograph and certain assumptions to construct two frequency curves for DT, one for tones and another for broadband signals, both at a single operating level. Andrews and Hovater (1971) took Barger's concept a step further and introduced the concept of sequential observations.

This report, by necessity, contains an outline of the more applicable points of the theory which led to the nomograph of Watters and Moore. It also considers in detail the assumptions and methods applied by Barger, Andrews, and Hovater. In some areas, the present study disagrees with the latter two reports. In those cases, alternatives are presented which appear more realistic. In still other areas, new material is presented which was not considered in these first attempts at a quantitative model. The points of difference should not be taken as a criticism of the methods previously used, but should rather serve to improve the model and to point to areas where further research might eliminate uncertainties.

CHAPTER 2

STATISTICAL DECISIONS AND SIGNAL DETECTABILITY

The theory of signal detectability as it has emerged in the field of psychoacoustics is based on the pioneering work of Peterson, Birdsall, and Fox (1954). The problem is to distinguish between a background of noise with an embedded signal, and the noise alone. If the statistics of the noise (N) and of the signal plus noise (SN) are known, one can predict the optimum behavior of any detection system. This optimum performance can be expressed as a Receiver Operating Characteristic, or ROC. Swets, Tanner, and Birdsall (1961) have provided an excellent article explicating the theory and describing experiments that constitute the primary tests of its validity. While their article is concerned with visual detection, a somewhat less pedantic report by Tanner, Swets, and Green (1956) shows, through similarly designed experiments, that the theory provides a valid description of aural detection as well. Some of the highlights of the theory are outlined here.

The probability density functions for an observation x , for the cases when x is drawn from populations N and SN, are defined as $f_N(x)$ and $f_{SN}(x)$, respectively.* In general, these two densities might appear as shown in Figure 1. Depending on the a priori probability of the signal being present, values and costs of correct and incorrect decisions, and the densities $f_N(x)$ and $f_{SN}(x)$, an operating point c for the receiver is determined. For any observation $x > c$ the signal is considered present; for any observation $x < c$ it is considered absent.

From Figure 1 it can be seen that, regardless of the position of the operating point, observations $x > c$ can be caused by either the presence of the signal plus noise (a true detection) or by noise alone (a false alarm). The probability that $x > c$ comes from $f_{SN}(x)$ is called the probability of detection and is given by

$$P(D) = \int_c^{\infty} f_{SN}(x) dx. \quad (2-1)$$

* x can be multidimensional, i.e., $x = (x_1, x_2, x_3, \dots)$

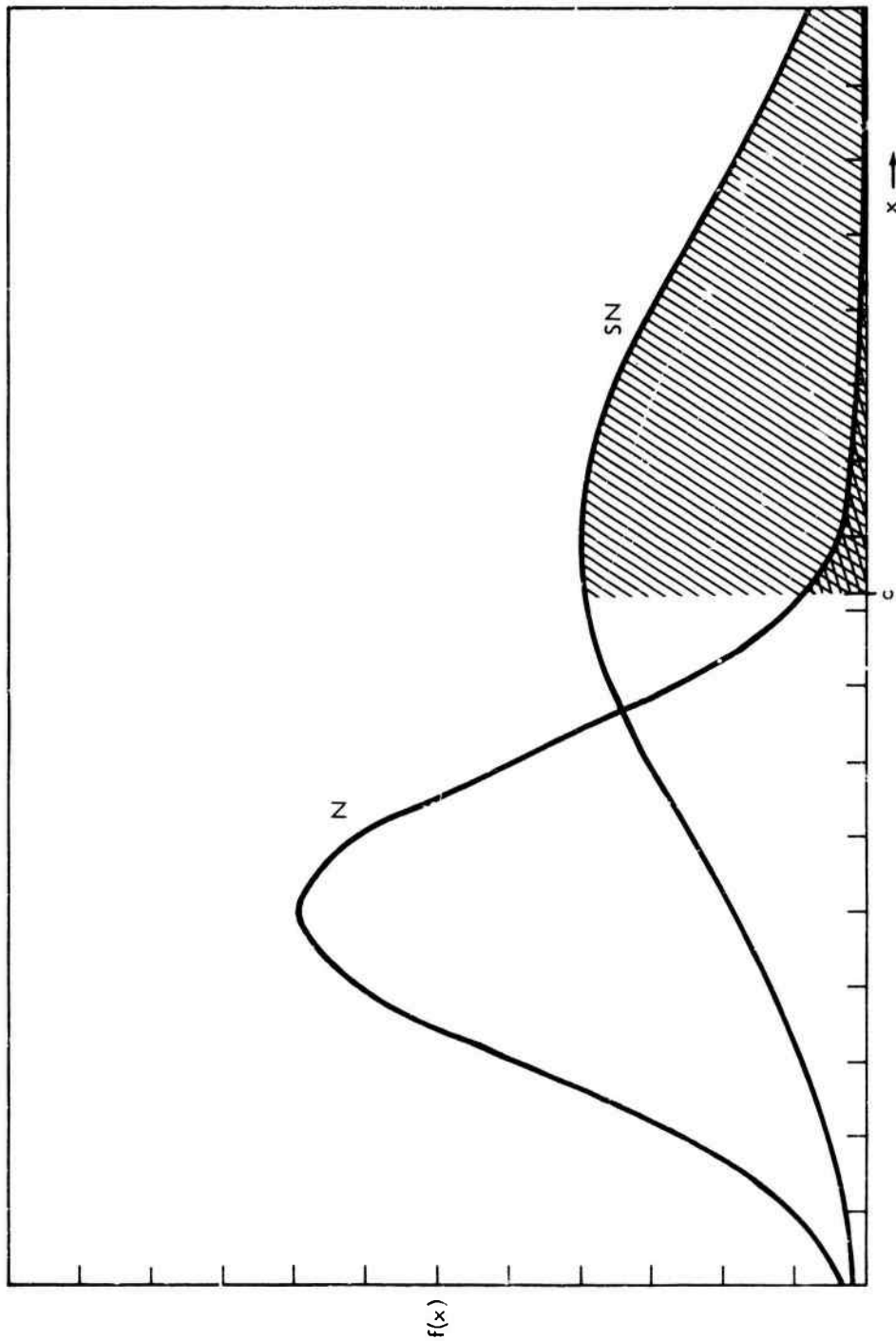


FIG. 1. HYPOTHETICAL VALUES FOR $f_N(x)$ AND $f_{SN}(x)$

The cross hatched area below the SN curve in Figure 1 represents $P(D)$ graphically. Likewise, the probability that $x > c$ comes from $f_N(x)$ is called the probability of false alarm and is given by

$$P(FA) = \int_c^{\infty} f_N(x) dx. \quad (2-2)$$

The double cross hatched area below the N curve in Figure 1 represents $P(FA)$ graphically. In general,

$$\int_{-\infty}^{\infty} f(x) dx = 1. \quad (2-3)$$

A ROC curve is defined on a graph of $P(D)$ vs $P(FA)$ by letting the operating point c in Figure 1 vary from minus to plus infinity (see Figure 2). If either probability density changes shape or if their relative separation changes, a new and different ROC curve is defined. The parameter d' in Figure 2 will be defined later in Equation (2-16).

The optimum operating point can be determined in the following manner. Let the probability of a miss be given by

$$\begin{aligned} P(M) &= \int_{-\infty}^c f_{SN}(x) dx \\ &= 1 - P(D). \end{aligned} \quad (2-4)$$

If the a priori probability of the signal being present is P_{SN} and the a priori probability of the signal being absent is P_N , then of course,

$$P_{SN} + P_N = 1. \quad (2-5)$$

An incorrect decision can be made in two ways, i.e., the observer can decide a signal is present when it is not, or he can decide a signal is not present when it is. The probability of error, then, is given by

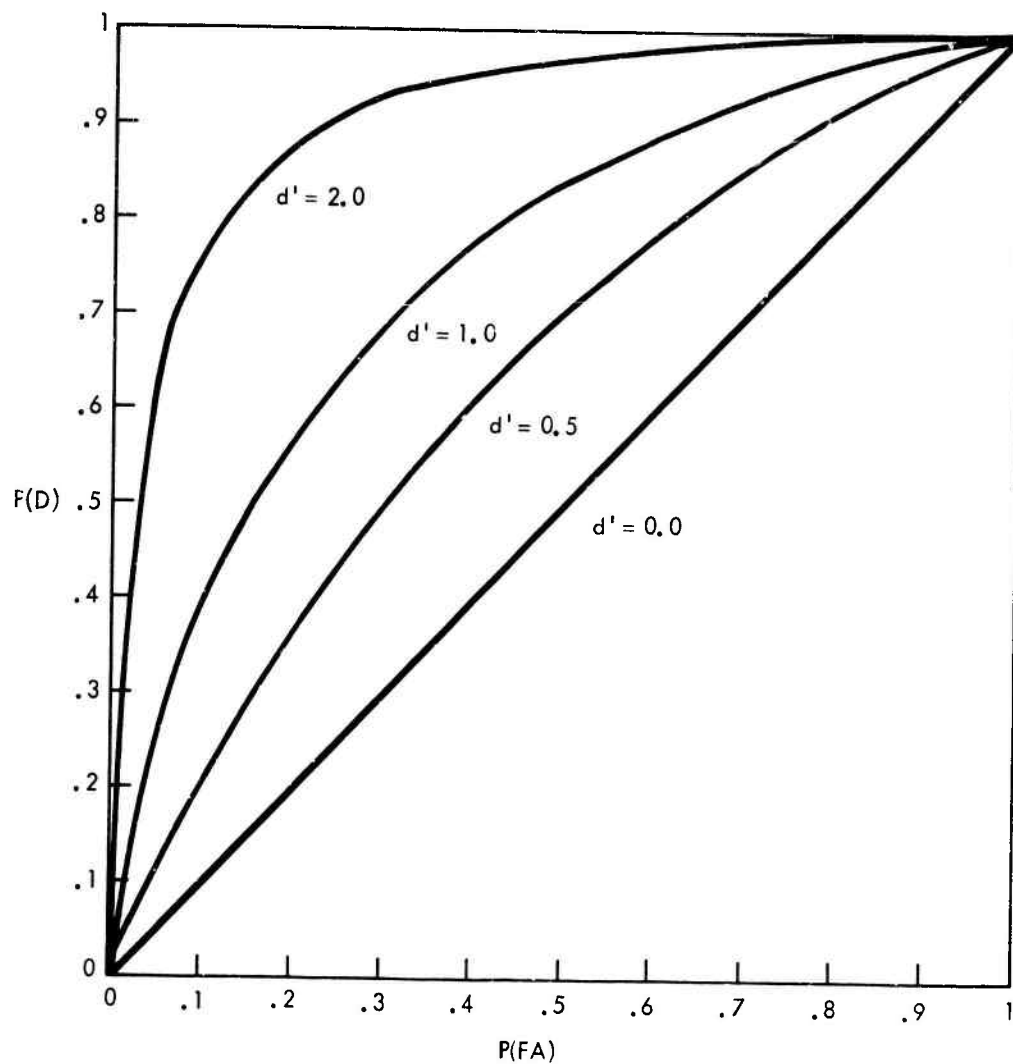


FIG. 2. ROC CURVES ASSUMING NORMAL DENSITIES WITH EQUAL VARIANCE

$$P(E) = P_N \cdot P(FA) + P_{SN} \cdot P(M) \quad (2-6)$$

$$\begin{aligned} &= (1 - P_{SN}) \cdot P(FA) + P_{SN} \cdot [1 - P(D)] \\ &= P_{SN} \cdot \left\{ 1 - \left[P(D) - \frac{(1 - P_{SN})}{P_{SN}} \cdot P(FA) \right] \right\} \end{aligned}$$

It is desired to minimize $P(E)$. According to Equation (2-6), this criterion is equivalent to maximizing the term in the square brackets. That is,

$$P(D) - \frac{(1 - P_{SN})}{P_{SN}} \cdot P(FA) = \text{maximum}. \quad (2-7)$$

For convenience define β as the coefficient of $P(FA)$ in Equation (2-7). Equation (2-7) is satisfied when the first derivative is zero. That is,

$$\frac{dP(D)}{dc} - \beta \cdot \frac{dP(FA)}{dc} = 0. \quad (2-8)$$

Rearranging,

$$\beta = \frac{dP(D)}{dc} \bigg/ \frac{dP(FA)}{dc}. \quad (2-9)$$

Equation (2-9) reduces after some algebra* to

$$\beta = \frac{f_{SN}(c)}{f_N(c)}. \quad (2-10)$$

*Leibnitz's rule (Wylie, 1966) says that if $F(t) = \int_{a(t)}^{b(t)} \phi(x, t) dx$, where a and b are differentiable functions of t , and where $\phi(x, t)$ and $\frac{\partial \phi(x, t)}{\partial t}$ are continuous in x and t , then $\frac{dF}{dt} = \int_{a(t)}^{b(t)} \frac{\partial \phi(x, t)}{\partial t} dx$

+ $\phi[b(t), t] \frac{db(t)}{dt} - \phi[a(t), t] \frac{da(t)}{dt}$. In our case, $P(D) = \int_c^\infty f_{SN}(x) dx$. Therefore, $\frac{dP(D)}{dc} = \lim_{\alpha \rightarrow \infty} \left[\int_c^\alpha \frac{\partial f_{SN}(x)}{\partial c} dx + f_{SN}(\alpha) \frac{d(\alpha)}{dc} - f_{SN}(c) \frac{dc}{dc} \right] = -f_{SN}(c)$.

Similarly $\frac{dP(FA)}{dc} = -f_N(c)$.

The optimum operating point is defined, then, such that when the likelihood ratio

$$\lambda = \frac{f_{SN}(x)}{f_N(x)} \quad (2-11)$$

is greater than or equal to β , the signal is considered present.

The above definition of β assumes that values and costs of correct and incorrect decisions are equal. In general, β is given by

$$\beta = \frac{1 - P_{SN}}{P_{SN}} \cdot \frac{C_{22} - C_{21}}{C_{11} - C_{12}} \quad (2-12)$$

where C_{11} is the value of a correct positive report, C_{12} is the value of an incorrect negative report, C_{21} is the value of an incorrect positive report, and C_{22} is the value of a correct negative report (Harman, 1963; Green and Swets, 1966).

So far nothing has been said about the exact shapes of $f_N(x)$ and $f_{SN}(x)$. The usual assumption (Green and Swets, 1966) made is that they are both Gaussian with equal variances. That is,

$$f_N(x) = \frac{1}{\sqrt{2\pi}\sigma_N} \exp \left[-1/2 \left(\frac{x - \mu_N}{\sigma_N} \right)^2 \right] \quad (2-13)$$

where μ_N and σ_N are the mean and standard deviation, respectively, of $f_N(x)$,

$$f_{SN}(x) = \frac{1}{\sqrt{2\pi}\sigma_{SN}} \exp \left[-1/2 \left(\frac{x - \mu_{SN}}{\sigma_{SN}} \right)^2 \right] \quad (2-14)$$

and

$$\sigma_N = \sigma_{SN} = \sigma. \quad (2-15)$$

If the densities are normal, the ROC curves for the ideal receiver will be straight lines with positive slope, when plotted on normal-normal paper. If the standard deviations are equal, the linear ROC curves will have a positive slope of unity.

A convenient single parameter called the detectability index which describes the relative separation between the two densities in terms of the standard deviation is

$$d' = \frac{\mu_{SN} - \mu_N}{\sigma_N} . \quad (2-16)$$

Note that a given value of d' specifies a particular ROC curve as shown in Figure 2. Figure 3 shows ROC curves for various d' plotted on normal-normal paper. For convenience, at the top and right of Figure 3, the graph is scaled in terms of the normal deviate, K_N and K_{SN} . Thus, rearranging Equation (2-16) and using Equation (2-15),

$$\begin{aligned} d' &= \frac{c - \mu_N}{\sigma_N} + \frac{\mu_{SN} - c}{\sigma_{SN}} \\ &= K_N + K_{SN} . \end{aligned} \quad (2-17)$$

Peterson, et al., show that for a sinusoid of known phase in Gaussian noise, i.e., when the signal is known exactly (SKE), the assumptions of Equations (2-13) through (2-15) hold, with Equation (2-16) becoming

$$d' = \sqrt{\frac{2E}{N_0}} \quad (2-18)$$

for the ideal receiver, where E is the signal energy and N_0 is the noise power per unit bandwidth.* Peterson, et al., also show that if the signal itself is Gaussian, i.e., the signal is known statistically (SKS), the same ROC curves apply for the ideal receiver if

$$d' = \frac{S}{N} \sqrt{WT} = \frac{E}{N_0 \sqrt{WT}} , \quad (2-19)$$

*A tutorial derivation of Equation (2-18) is given by Elliot (1959).

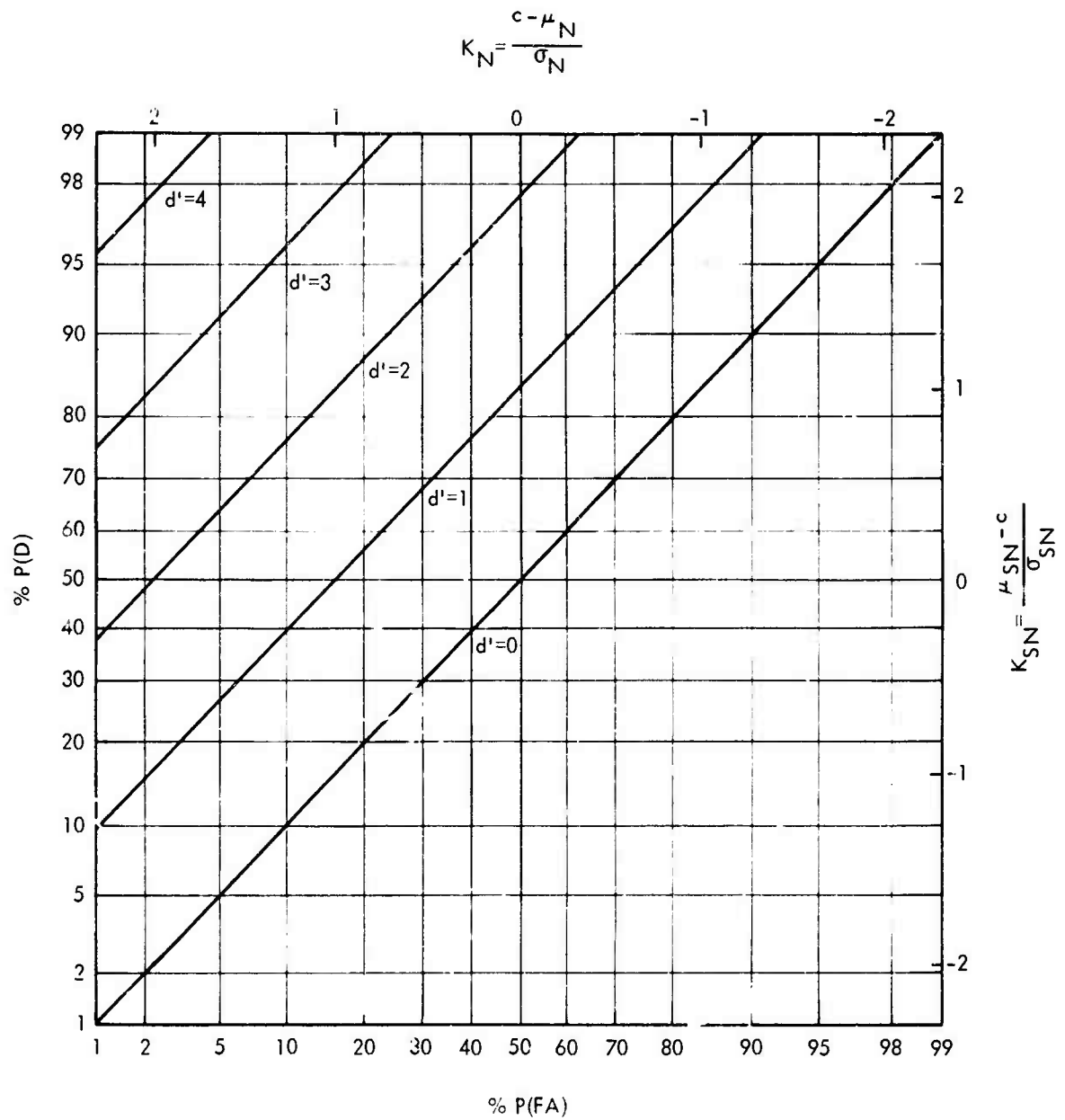


FIG. 3 ROC CURVES ON NORMAL-NORMAL PAPER ($d' = K_N + K_{SN}$).

where S is the signal power, N is the noise power in the signal band, W is the signal bandwidth, and T is the signal duration.[†] However, Equation (2-15) and thus Equation (2-19) are only valid when

$$\frac{S}{N} \ll 1. \quad (2-20)$$

Otherwise the expression for d' in terms of energy is somewhat more complicated and the resulting ROC curves are lowered, having slopes less than unity.

Equations (2-18) and (2-19) define d' for the ideal detector in terms of the signal energy for two specific cases of interest in the sonar problem. Similar relationships can be derived for the human hearing mechanism. That is, the choosing of a suitable operating level for the sonar operator will define a d' which in turn will determine the signal energy that is just detectable to the human ear at the chosen operating level. This result can then be plugged into the sonar equation.

In the next chapter, some of the experiments are reviewed which test the validity of the theory of signal detectability as a description of the auditory process. These experiments include the relationships between d' and signal energy which led to the nomograph of Watters and Moore.

[†] Equation (2-19) is derived by equating the lower limit of the integral in Equation (2-1) for SKE to that for SKS; then likewise for Equation (2-2). This procedure yields two equations in d' and a parameter related to signal energy.

[‡] The common logarithm of Equation (2-19), i.e., $10 \log \left(\frac{E}{N_0 \sqrt{WT}} \right)$, is known as the psychometric function.

CHAPTER 3

SIGNAL DETECTABILITY AND PSYCHOACOUSTICS

The primary test of the theory of signal detectability as a tool to describe aural detection is to determine if the ROC curves similar to those described in Chapter 2 adequately describe the performance of the hearing mechanism. Tanner, et al. (1956), sought to determine experimentally the shape of the ROC curve in a simple psychoacoustic task. The signal was a 1 kHz sinusoid of 0.1 sec duration. The masking stimulus was broadband Gaussian white noise. The noise and signal levels were held constant throughout the experimental session. A yes-no (YN) experiment* was conducted at five different a priori probabilities of the signal being present. Two subjects were informed of this a priori probability and were given immediate information after each response as to whether or not the signal was actually present. They were paid for a correct answer, fined an equal amount for an incorrect answer, and instructed to make as much money as possible. In this way, the parameters of Equation (2-12) were varied over a considerable range. The resultant ROC curve was best described by a straight line with slope slightly less than unity. These results would indicate that decision making theory provides an adequate description of the auditory process. The straight line on normal-normal paper verifies the assumption of normality. The slope slightly less than unity typifies the situation where σ_{SN} increases with increasing μ_{SN} .

The experiment by Tanner, et al. (1956), did not attempt to measure the relationship between d' obtained experimentally and the signal energy. It merely established that the theory of signal detectability adequately describes the auditory process. In general, the actual value of the detectability index d' might be derived in two ways. It could be defined by the signal and noise levels and characteristics of the experiment through Equations (2-18) or (2-19), or it might be measured by finding experimentally the various combinations of detection rates and false alarm rates, and then applying Equation (2-17). As will be shown, the more useful approach is the latter, since the ear is not a perfect detector and the relationships between d' and E are not quite those of Equations (2-18) and (2-19).

*In the yes-no (YN) experiment, the signal is either present or absent in a single observation interval. In the n alternative forced choice (nAFC) experiment, the signal appears in one and only one of n successive observation intervals.

Other investigations have provided additional evidence that the theory is correct while studying the effect of experimental procedure on the detectability index. Tanner, et al., also conducted a four alternative forced choice (4AFC) experiment with the other experimental variables the same as for the YN experiment described above. The experimentally determined values for d' from both methods were in close agreement, with the YN value falling slightly below the FC value. In a later study by Swets (1959), three observers participated in YN, 2AFC, and 4AFC experiments, each at various signal energy levels. Again, the signal was a 1 kHz tone with 0.1 sec duration. The estimates of d' vs. signal energy from the 2 and 4AFC experiments were nearly identical. The YN values were very similar to, though more variable than, the FC values. Swets also reported results for 2, 3, 4, 6, and 8AFC experiments at one signal energy level. For three different observers, a graph of the percentage of correct decisions $P(c)$ vs. the number of alternatives was nicely represented by a constant d' curve.* Egan, Schulman, and Greenberg (1959), obtained ROC curves using a YN procedure in which the observers were allowed to rate their yes answers. The signal was a 1 kHz pure tone of 0.5 sec duration. The results were straight lines with slopes slightly less than unity. Schulman and Mitchell (1966), compared ROC curves obtained from YN and 2AFC procedures. They allowed the observers to use a six point scale of confidence ratings for each observation with each method. The signal in each case was a 1 kHz tone of 0.1 sec duration. Their results show that linear ROC curves provide an adequate description of the data. However, the operating characteristics for the 2AFC procedure were clearly higher than those of the YN procedure and had unit slopes. The YN curves tended to have slopes less than unity.

The reason why the ROC curve for the YN experiment will inherently lie below that of the FC experiment can be seen by considering Equation (2-10) in the light of human behavioral processes. For simplicity assume that Equations (2-13) through (2-15) are valid with $u_N = 0$ and $c = 1$. Then

$$\begin{aligned} \beta &= \frac{\frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} (c-d')^2 \right]}{\frac{1}{\sqrt{2\pi}} \exp \left(-\frac{1}{2} c^2 \right)} \\ &= \exp \left[d' \left(c - \frac{1}{2} d' \right) \right]. \end{aligned} \quad (3-1)$$

*The relationships between d' and $P(c)$ for forced choice experiments are tabulated by Elliot (1959).

Figure 4 shows $f_N(x)$, $f_{SN}(x)$, and β under the above conditions. The observer tends to modify too extreme a judgement criterion. If β is set at a very large value, the observer is required to have an overwhelmingly clear indication of the presence of a signal before he ought to call it a signal. In practice, he will instinctively move the criterion downward and give a position indication of the presence of a signal if, for example, he is 99% sure the signal is present even though the value of β calls for being 99.9% sure. This tendency of most observers to relax the criterion effectively leads to a $\beta < \beta_{\text{optimum}}$. On the other hand, β small calls for a positive answer a distasteful proportion of times. That is, the observer should answer positively unless the absence of the signal is a near certainty. Most observers will tend to move the operating point upward, leading to $\beta > \beta_{\text{optimum}}$. In practice, the general trend in the YN experiment is to move in the direction toward $\beta = 1^*$ or $P_{SN} = 0.5$ in Equation (2-12) with unit cost matrix. Equation (2-9) says β is the slope of an appropriate ROC curve in Figure 2. It can be seen, then, that if β is very large, reducing β means going to a curve with a smaller slope which can be done only by operating on the lower ROC curve in Figure 2. Likewise for small (less than unity) β , $\beta < \beta_{\text{optimum}}$ implies a large slope, again lowering the ROC curve or decreasing performance. These behavioral processes do not apply in the FC case, where the problem is merely to decide which slot has the signal, not whether a signal appeared at all in the particular test.

It should be noted here that the detection of an underwater acoustic signal by a passive sonar operator is a YN procedure while available data shown later in Tables 3-1 and 3-2, relating d' to signal energy, have been taken using the apparently superior FC method. It is impossible to predict an exact relationship between d'_{FC} and d'_{YN} . The qualitative argument in the previous paragraph predicts a trend but says nothing about the magnitude of the effects at low and high values of β . It must be assumed that behavioral processes will vary from observer to observer with the result that the distance between the two curves as well as the slope of the d'_{YN} curve will vary. However, a reasonable engineering approximation which circumvents this difficulty is presented in the next chapter.

Once the theory has been shown adequate, and relationships among various experimental procedures are determined, the next step is to determine empirically the relationship between d' and signal energy for the hearing mechanism. Green, Birdsall, and

*Green (1960b) presents a graph of β -experimental vs. β -optimum for a YN experiment. The resulting straight line has slope less than unity and crosses the ideal curve at $\beta = 1$.

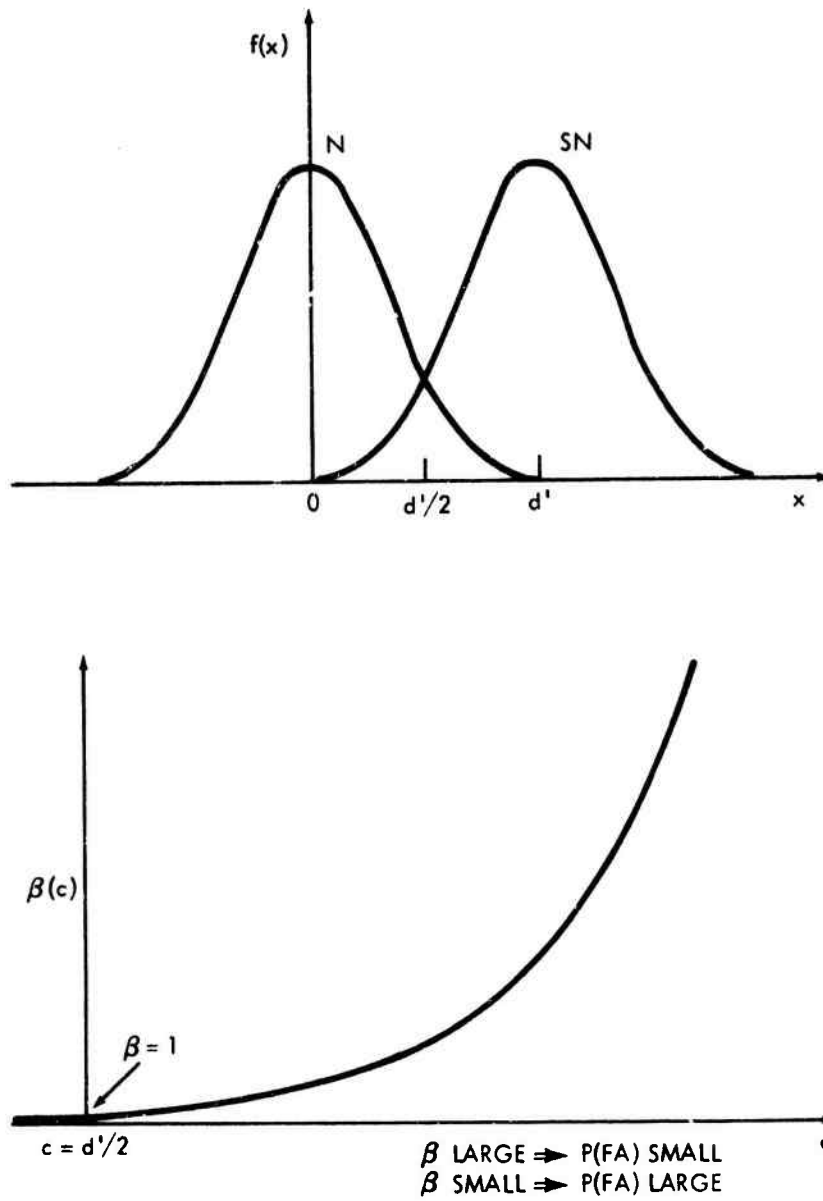


FIG. 4. OPTIMUM LIKELIHOOD RATIO

Tanner (1957) investigated the dependence of signal detectability on signal intensity and duration. Again, the signal was a 1 kHz tone in a background of broadband Gaussian white noise. Their experiment brought three important points to light. For human observers d' varies linearly with signal energy, or more like Equation (2-19).^{*} Secondly, there is a limit to signal duration beyond which the human hearing mechanism does not integrate the signal perfectly. Third, there is an inherent inefficiency to the hearing mechanism, i.e., Equation (2-19) must be multiplied by a constant to predict experimental results. The latter two results might have been expected due to the finite capacity of the human brain to process information. The linear variation of d' with signal energy for the case of a pure tone in Gaussian noise was more surprising in the light of Equation (2-18). Tanner and Birdsall (1958) present a plausible explanation for this linear variation. Their arguments lead to the conclusion that if particular information such as signal phase, etc., is not utilized by the receiver being studied, the results are the same as if uncertainty in that particular signal property had been introduced at the transmitter. If one accepts this reasoning, the application of the SKS model in the case of aural detection of a sinusoidal signal in Gaussian noise seems reasonable.

Green, McKey, and Licklider (1959) made further investigations designed to quantify the relationship between d' and E . In a 2AFC experiment, they found the efficiency "constant" to be, in fact, a function of frequency as well as d' . The signal was a tone of 0.1 sec duration at various frequencies and signal energies. The noise stimulus was broadband Gaussian white noise. Their results for three values of d' are given in Table 3-1. Their range of frequencies was 250 to 4000 Hz over a wide range of d' .

d'	$10 \log (E/N_o)$	(db)
.54	$5.6 + 2.0 f$	(kHz)
.95	$7.4 + 2.0 f$	
1.47	$9.1 + 2.0 f$	

Table 3-1. Experimental Data for Detectability Index vs. Psychometric Function for a Sinusoid

^{*}For a pulsed sinusoid the time - bandwidth product is unity.

All the previously described work was concerned with the detection of a pure tone masked by noise. Green (1960a) measured the auditory detection of a noise signal in Gaussian noise. In a 2AFC experiment, he found the efficiency to be constant for a given d' , for various combinations of signal duration, bandwidth and center frequency. Moreover, he found that the bandwidth of the masking noise made little difference so long as it overlapped the signal width. The efficiency did turn out to be a function of d' . Green's data for three values of d' are given in Table 3-2.

d'	$10 \log \left(\frac{E}{N_0 \sqrt{WT}} \right) \text{ (db)}$
.36	3.5
.95	5.0
1.81	6.4

Table 3-2. Experimental Data for Detectability Index vs Psychometric Function for a Gaussian Signal.

Tables 3-1 and 3-2 give values for the psychometric function at particular values of d' . Least square fits to the experimental data of Green, et al. (1959), and Green (1960a), are plotted in Figure 5.* The psychometric function is labeled $A(d')$ for convenience, i.e.,

$$A(d') = 10 \log \left(\frac{E}{N_0 \sqrt{WT}} \right) . \quad (3-2)$$

The sought after relationship between detectability index and signal energy for human aural detection is given in Figure 5. There remain to consider in the following chapter some of the differences between the closely controlled laboratory experiment and detection by passive sonar in the ocean.

*Figure 5 differs somewhat from the nomograph of Watters and Moore (1970). Dr. Moore indicated in a telephone conversation that his curves were to have been corrected for time uncertainty in the detection interval. Inadvertently, these corrections were applied only to the data of Green (1960a).

CHAPTER 4

PSYCHOACOUSTICS IN THE LABORATORY AND THE REAL WORLD

Aural detection in the controlled laboratory experiments discussed here and the detection of an underwater acoustic source by passive sonar are problems that exist in two different worlds. The psychoacoustic test involves the detection of either pure tone bursts of constant frequency, amplitude and duration, or pure Gaussian noise of constant bandwidth, center frequency, average power and duration, in a background of pure Gaussian noise of constant bandwidth, center frequency and amplitude. In either laboratory case, the signal is completely specified to the observer before the experiment begins, P_{SN} is known, and the precise time interval for the process is defined.

On the other hand, a real ship or torpedo signature is a mixture of many lines of varying frequency and amplitude superimposed on a continuous broadband signal of varying spectral shape and amplitude. The masking background is at best broadband noise of varying spectral shape and amplitude and often has line character of its own. Neither the a priori probability of the signal being present nor the time of its possible presence are known. What are the effects of these and other differences? Answers to this question are discussed below.

The first difficulty is the choosing of an operating level. While the values and costs of correct and incorrect decisions can be estimated, a knowledge of P_{SN} is another matter. Certainly in the case of torpedo detection, for example, P_{SN} is small even in the time of war unless the target has already detected the presence of the firing ship. From Equation (2-12), when P_{SN} is small, β becomes large. As was pointed out in Chapter 3, for large β the human tends to choose an operating point such that performance is less than optimum. While an operator who has been alerted might perform better, due to a higher P_{SN} (or β closer to unity), there is still no quantitative way to determine the optimum operating point. However, this does not preclude the usefulness of the method. We merely live with less than optimum performance and an inherent variability due to differences in behavioral process from observer to observer. Andrews and Hovater (1971) estimate that reasonable results for humans can be obtained with signal to noise ratios such

that $0.4 \leq d' \leq 2.0$. Table 4-1 gives various combinations of $P(D)$ and $P(FA)$ for d' in this range, assuming Equations (2-13) through (2-15) hold. It should be pointed out that $P(D)$ and $P(FA)$ are defined for relatively short intervals, on the order of 1 sec, and not for the total duration of the encounter with the ship or torpedo. The extension of these concepts to total encounter times is discussed in Chapter 5.

$d' = 0.5$		$d' = 1.0$		$d' = 1.5$		$d' = 2.0$	
$P(D)$	$P(FA)$	$P(D)$	$P(FA)$	$P(D)$	$P(FA)$	$P(D)$	$P(FA)$
0.69	0.50	0.84	0.50	0.93	0.50	0.98	0.50
0.66	0.46	0.79	0.42	0.88	0.38	0.95	0.34
0.62	0.42	0.73	0.34	0.82	0.27	0.88	0.21
0.58	0.38	0.66	0.27	0.73	0.18	0.79	0.12
0.54	0.34	0.58	0.21	0.62	0.12	0.66	0.05
0.50	0.31	0.50	0.16	0.50	0.07	0.50	0.02

Table 4-1. Various Combinations of $P(D)$ and $P(FA)$ for a Given Value of d' .

Yes-No vs Forced Choice

Once the operating level is determined, a useable relationship between d'_{YN} and d'_{FC} must be ascertained so that the FC data of Figure 5 may be used. Schulman and Mitchell (1966) show theoretically and verify experimentally that

$$D_{FC} = \sqrt{2} D_{YN} , \quad (4-1)$$

where D_{FC} and D_{YN} are the perpendicular distances from the point $P(D) = P(FA) = 0.5$ to the linear ROC curves (plotted on normal-normal paper) obtained from the two procedures using identical boundary conditions. While the slope of the YN curve will vary from observer to observer, a reasonable approximation would assume that this slope is unity. The result is that

$$d'_{FC} \approx \sqrt{2} d'_{YN} \quad (4-2)$$

or the detectability in the YN procedure is decreased by a factor of $\sqrt{2}$.

Slowly Increasing Signal

When the signal is slowly increased, as is the case in passive sonar detection, the minimum detectable signal is greater than if the signal is pulsed. This is especially true if the signal and the noise are of the same general character where the only detectable change is one of magnitude. Horton (1959) estimated the decrement to performance due to this effect at about 3 db. No detectability index was specified since the theory of signal detectability was not used then.

Fluctuating Signal

Urick and Gaunaud (1972) show that fluctuations cause the detectability of weak signals to be higher. This phenomenon can be visualized as giving the signal a "pulsed" character. They define a fluctuation index k as

$$k = \left[1 + \left(\frac{\sigma_M}{\sigma_N} \right)^2 \right]^{1/2} \quad (4-3)$$

where σ_M is the standard deviation of the fluctuations of the signal. The fluctuation index can be related to V , the coefficient of variation in amplitude of the signal by

$$V = \frac{\sigma_M}{\mu_{SN}} \quad (4-4)$$

That is, if μ_N is set equal to zero, in Equation (2-16),

$$k = [1 + (d'V)^2]^{1/2} \quad (4-5)$$

In the limit of large distances, the distribution of fluctuations is Rayleigh (Skudrzyk, 1957), so that the coefficient of variation in amplitude given in Equation (4-4) is numerically equal to 0.52.

Whitmarsh (1963) gives an experimental value of 0.14 at 3 kyds. This is not inconsistent with Skudrzyk if the effect is roughly proportional to distance, and large distances are assumed to mean at least 10 kyds. Then, from Equations (4-3) and (4-5),

$$\frac{\sigma_M}{\sigma_N} \approx 0.52 d'. \quad (4-6)$$

For fluctuations of magnitude $n\sigma_M$, which have period of the order of the integration time of the ear, Equation (4-6) predicts that, for a given $P(FA)$, $P(D)$ is increased such that

$$d'_{FP} \approx (1 + 0.52 n) d'_{FNP}, \quad (4-7)$$

where FP and FNP stand for fluctuations present and not present, respectively.

Uncertain Signal Frequency

There have been many studies of the effect of uncertainty in signal frequency (Tanner, et al., 1956; Veniar, 1958; Swets, Shipley, McKey and Green, 1959; Creelman, 1960; Green, 1961; and Gundy, 1961). In general, the experiments first established a baseline for comparison by recording the performance when the signal frequency was specified. All the studies reveal that performance decreases as the number and range of possible signal frequencies increase. Green's study in particular produced consistent quantitative results. He found the decrease in detectability of a gated sinusoidal signal in noise to be about 3 db at $d' = 1$, in the case where the range of signal frequency uncertainty was 3.5 kHz. (The signal occurred anywhere between 500 and 4000 Hz). His results took into consideration the variation of the psychometric function with frequency.

Time Uncertainty

Egan, Greenberg, and Schulman (1961) and Egan, Schulman, and Greenberg (1961) investigated the effect of time uncertainty in the presentation of the signal. They found that the performance decreased as the uncertainty in the starting time increased. For various values of signal energy, they found the limit to the effect to be

$$d'_{STS} = 2 d'_{STU} , \quad (4-8)$$

where STS and STU stand for starting time specified and unspecified, respectively. This limit value occurred for values of starting time uncertainty between 4 and 8 sec.

Combination of Effects

Up to this point, five factors have been discussed which would modify the relationships of Figure 5 for simple pulsed signal tests to a passive sonar situation. The problem now is to combine their effects. Green (1960b) in a discussion following the ideas of Peterson, et al. (1954), argues that the uncertainties in the signal do not add directly. His model is that of an observer faced with the task of detecting a signal about which there are M orthogonal uncertainties. As Green indicates in a plot of psychometric function A(d') vs. M,

$$\frac{dA(d')}{dM} > 0 \quad (4-9)$$

but

$$\frac{d^2 A(d')}{dM^2} < 0. \quad (4-10)$$

That is, as M increases, its effect on A(d') decreases. Green's graph indicates that uncertainty M has half the effect of uncertainty M-1.

It is difficult to determine exactly how all the above listed uncertainties affect the detectability index when they are combined in the real world situation of detection by passive sonar in the ocean without conducting an exceedingly complex actual experiment. However, the following arguments are presented as a reasonable way to proceed.

First, consider the opposing effects of a slowly increasing signal and fluctuations of that signal. Horton gave a value of 3 db less detectability for a slowly increasing, or non-pulsed signal. Fluctuations can be shown to increase the detectability by about the same amount in the following way. Assign the parameter n in Equation (4-7) a value of 1.7 (1.7 σ_M will be exceeded 5% of the time assuming a Gaussian distribution), and d' a value of 2

(the upper limit of the interval $0.4 \leq d' \leq 2.0$ given by Andrews and Hovater (1971) as reasonable). Then Equation (4-7) says that for d' of 2.0 with fluctuations present as in the sonar case, the d' for fluctuations not present must be divided by a factor of 1.9, yielding 1.1. Figure 5 shows that the db difference along a typical curve such as for 1 kHz between d' of 2.0 and 1.1 is 2.8 db. Since these two effects are of the opposite sign and of the same approximate magnitude, and since no detailed experimental evidence as to their exact effects exist, they are assumed to cancel.

Secondly, it is assumed that the effect of frequency uncertainty, measured at $d' = 1$ to be 3 db, varies as does the effect of time uncertainty. Since the effect of time uncertainty is to halve the detectability index and since at $d' = 1$ this is equivalent to 3 db (see change of $A(d')$ from $d' = 2$ to $d' = 1$ on typical curve of Fig. 5) as in the case of frequency, it is assumed that the effect of frequency uncertainty is also to halve the detectability index when considered alone. Therefore, since both time and frequency uncertainties, considered independently, halve the detectability index, we can write

$$\begin{aligned} d' &= (1 + 1) d'_F = (1 + 1) d'_T & (4-11) \\ &= 2 d'_F = 2 d'_T \end{aligned}$$

where d'_T is the detectability index for time uncertainty and d'_F is the detectability index for frequency uncertainty. When both effects are considered together in the light of Green's arguments concerning multiple orthogonal uncertainties, one gets

$$\begin{aligned} d' &= (1 + 1 + 0.5) d'_{FT} & (4-12) \\ &= 2.5 d'_{FT} \end{aligned}$$

where d'_{TF} stands for the effective detectability index with both time and frequency uncertainty and uncertainty $M + 1$ is assumed to have one half the effect of uncertainty M .

Finally, in order to use the FC data in Figure 5 in the YN passive sonar problem, Equations (4-2) and (4-12) are combined to yield

$$\begin{aligned} d' &= \sqrt{2} \cdot 2.5 d'_{\text{eff}} & (4-13) \\ &= 3.5 d'_{\text{eff}} \end{aligned}$$

where d'_{eff} is the effective d' in the passive sonar problem.

Equation (4-13) represents an inefficiency of the human ear in detecting a signal in a passive sonar situation, as contrasted to its ability to detect a pulsed signal in the n Alternative Forced Choice situation corresponding to Figure 5. What remains is to determine the correction to Figure 5. It has already been pointed out that both pure tones and broad band noise tend to be detected roughly in accordance with Equation (2-19). Figure 5 represents the best experimental modification to the theoretical relationship of Equation (2-19) which predicts that d' should double each time the signal energy doubles. Study of the slope of the pure tone curves in Figure 5 shows that they have on the average a slope of about 3 db change in $A(d')$ per doubling of d' , which agrees well with the linear relationship of Equation (2-19). The desired relationship for passive sonar must have the same slope for the curves relating d' to $A(d')$ as in Figure 5. The appropriate correction to Figure 5, therefore, is derived from Equations (2-19), (4-13), and (3-2) as

$$\begin{aligned} A(d') &= 10 \log 3.5 & (4-14) \\ &= 5.4 \text{ db.} \end{aligned}$$

Figure 6 has been derived from Figure 5 by shifting the curves over by 5.4 db to give the detectability index for the human ear in passive sonar detection. The data of Figure 6 are used in the following chapter to derive expressions

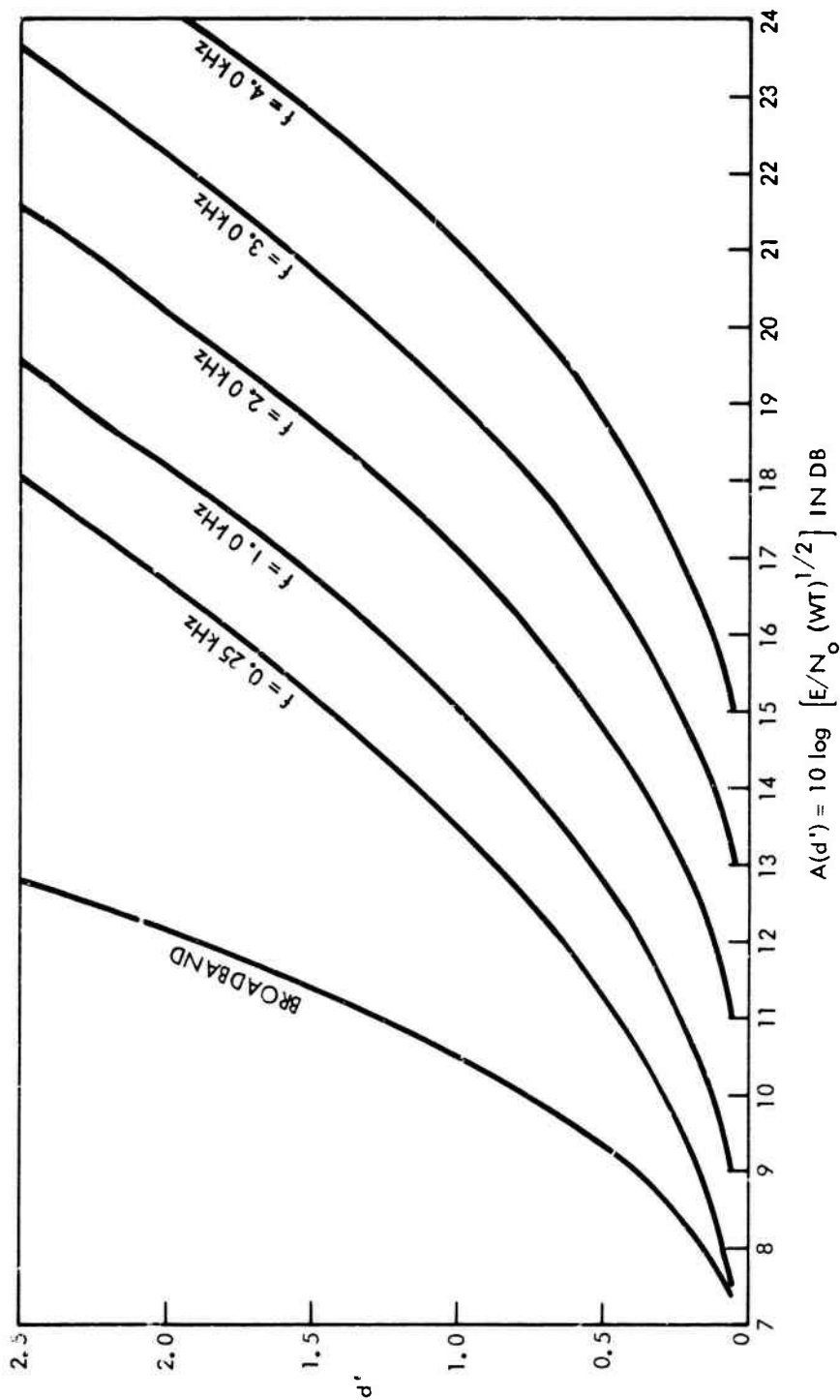


FIG. 6. DETECTABILITY INDEX FOR HUMAN EAR IN PASSIVE SONAR DETECTION

for DT as a function of frequency and type of signal. The ordinate in Figure 6 is really d'_{eff} from Equation (4-13), but the subscript has been dropped for the sake of simplicity through the rest of the discussion.

CHAPTER 5

DETECTION THRESHOLD AND THE SONAR EQUATION

As pointed out in Chapter 1, the sonar equation parameters are, in general, functions of frequency. Any complete discussion of the equation must, therefore, take frequency into account. Barger (1971) has presented a useful graphical treatment of the sonar equation. His method combines NL, DI, and DT in the form of an "Equal Detectability Curve," or EDC. The EDC is positioned over a plot of SL with the frequency scales lined up. Then the EDC is slid downward until it just touches the SL curve. The FOM is the difference between the ordinate values of the SL and EDC curves at the point of coincidence. Thus, the process yields the numerical solution of Equation (1-1). The point of intersection determines the frequency causing the detection. Barger's method thus facilitates the calculation of detection range and points to the frequency location where reductions in SL would have the greatest effect in decreasing FOM and consequently, detection range.

Barger (1971) presented two EDC's, one for tonal signatures and one for broadband signatures*. In the discussion below, DT is derived for lines and for broadband signatures, based on the discussion of Chapters 3 and 4, pointing out any variance with the method of Barger. The detection threshold for lines is defined as that ratio of signal to noise that can just be detected by the human ear, and is expressed in db as

$$DT = 10 \log \frac{S}{N_0} . \quad (5-1)$$

The expression for DT can be related to the psychometric function, $A(d')$, using Equation (3-2), and the relationships $E = ST$ and $WT = 1$. Thus

$$DT = A(d') - 10 \log T. \quad (5-2)$$

*A glance at Figures 5 and 6 show that DT is a function of the type of signal. The two types of signal must, therefore, be treated separately, using the appropriate value for DT.

For passive sonar detection, the non-transient signal is effectively a continuous one of unlimited extent so that the signal duration parameter, T , is irresolute. However, Green et al. (1957), found that there is an upper limit to T beyond which the performance of the hearing mechanism typifies imperfect integration. This value of T was estimated to be about 150 msec. Licklider and Green (1961) presented more thorough data pertaining to the integration time. They found for a 1 kHz tone that the psychometric function is flat for $T < 0.15$ sec, rises linearly with slope 1.5 db per doubling of time for $0.15 < T < 1.5$ sec, and rises linearly at the rate of 3 db per doubling of time for $T > 1.5$ sec.* These findings mean that the detection threshold decreases with slope 3 db per doubling of time for $T < 0.15$ sec, decreases with slope 1.5 db per doubling of time for $0.15 < T < 1.5$ sec, and is flat for $T > 1.5$ sec. Quantitatively this means that, for passive sonar detection of a continuous signal, the quantity $(10 \log T)$ must have a fixed value of

$$10 \log T + 10 \log (0.15) + 5 \log \frac{1.5}{0.15} \quad (5-3)$$

$$= -3.2 \text{ db.}$$

This corresponds to a limit on the fully effective integration time for the human ear of 0.48 sec. The expression for DT , then, is

$$DT = 3.2 + A(d'). \quad (5-4)$$

Equation (5-4), together with the data of Figure 6, produces the family of curves shown in Figure 7[†] for DT vs. frequency for various values of d' for detection of lines by passive sonar. The dashed portions of the curves are extrapolations since data were not taken at those frequencies.

*The actual data do not contain these sharp discontinuities, but the above description is an adequate one.

†Barger (1971) presented a curve for DT for lines using $d' = 1.7$ and the uncorrected values of $A(d')$ as reported by Watters and Moore (1970). For T he used the geometric mean of 0.15 and 1.5, which is equivalent to the procedure of Equation (5-3). His curve is approximately 5.4 db lower than Figure 7 would yield.

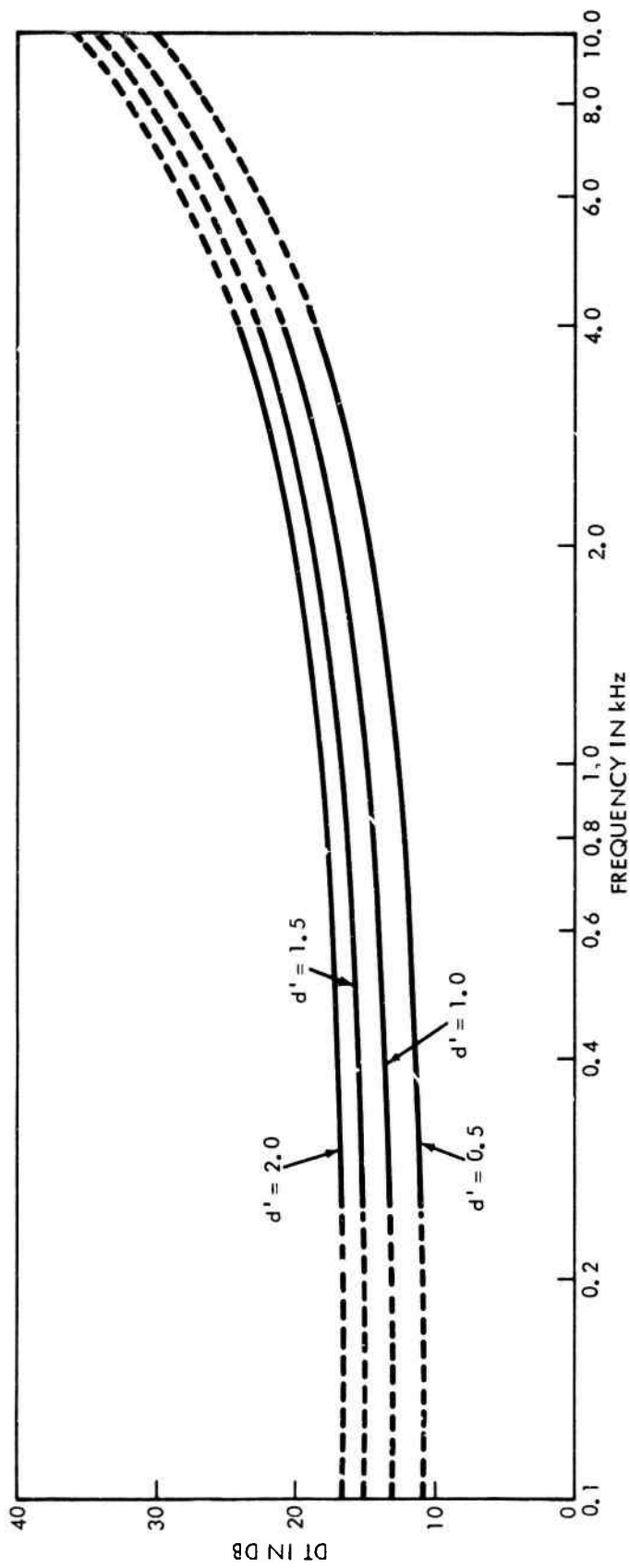


FIG. 7. DETECTION THRESHOLD FOR TONES IN PASSIVE SONAR DETECTION

The broadband signal is not so straightforward to treat. Here,

$$DT = 10 \log \frac{S}{N} , \quad (5-5)$$

and $WT \neq 1$, so that Equation (3-2) leads to

$$DT = A(d') - 5 \log W - 5 \log T. \quad (5-6)$$

If the integration times of the ear for broadband and tonal signals are identical*, Equations (5-3) and (5-6) lead to

$$DT = 1.6 + A(d') - 5 \log W, \quad (5-7)$$

where W is the width of the broadband signature in the detection region. A basic problem arises in the broadband detection case in that any EDC must be calculated for a specific bandwidth but there is no way of knowing the bandwidth of the signal, W , which will cause detection until after an EDC has been applied to the SL curve. For the common case where SL is presented in proportional bandwidths such as 1/3-octave (23%) or 1/30-octave (2.3%), it is useful to define

$$DT' = DT - 5 \log (af/W) \quad (5-8)$$

$$= 1.6 + A(d') - 5 \log (af),$$

where a is a constant and f is frequency, with units of both chosen to give af in Hertz. This artifice allows the handy calculation of FOM' , defined below. It does not require a prior knowledge of the frequency region causing detection. Equation (5-8) and the data of Figure 6 were used to produce the family of curves in Figure 8 which

*In a recent report, Moore (1972) would add 1 db to Equation (5-7), quoting the data of Green (1960a). We feel that Green's data is rather sketchy and could easily be interpreted to yield the same value for T as that of Licklider and Green (1961). We would, therefore, use the same value in both cases.

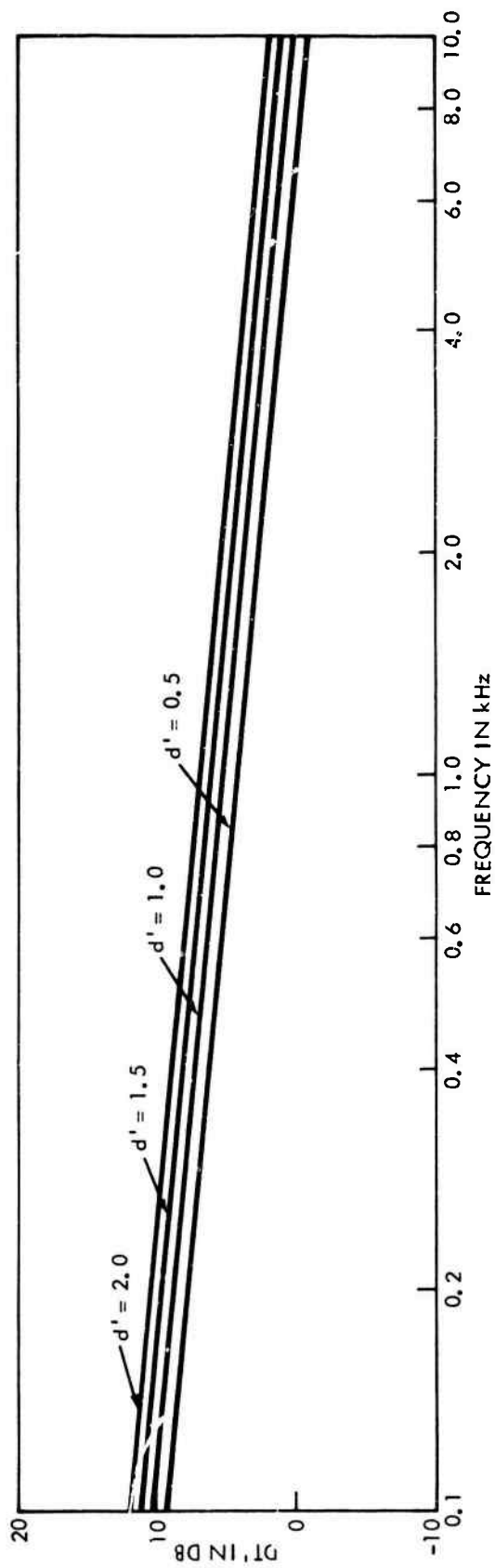


FIG. 8. DT' FOR BROADBAND SIGNALS IN PASSIVE SONAR DETECTION
($\sigma = 0.023$)

gives DT' vs. frequency for $a = 0.023$ and various values of d' . For constant bandwidth signatures, define

$$DT' = DT + 5 \log(W) \quad (5-9)$$

$$= 1.6 + A(d'),$$

which is constant for a given d' .

Corresponding to Equation (1-1), a similar expression can be set up using DT' . That is,

$$FOM' = SL - NL + DI - DT'. \quad (5-10)$$

Subtracting (5-10) from (1-1) gives

$$FOM - FOM' = DT' - DT. \quad (5-11)$$

Using Equations (5-7) and (5-8) for proportional bandwidth signatures

$$FOM = FOM' + 5 \log(W/af). * \quad (5-12)$$

Using Equations (5-7) and (5-9) for constant bandwidth signatures

$$FOM = FOM' + 5 \log(W). \quad (5-13)$$

For proportional bandwidth signatures, the procedure is to apply an EDC based on any value of a , since if a is changed, the shape of the curve is not changed, only its absolute level. Then after the general area of intersection with the SL curve (presented for the same bandwidth) is found, and a value of FOM' determined, a correction according to Equation (5-12) can be calculated.** For constant bandwidth signatures, apply an EDC based on Equation (5-9), and afterwards correct according to Equation (5-13).

*Since W in general will be greater than af , the value of FOM is increased over FOM' by $5 \log$ (number of analysis bands contained in the bandwidth of the signal contributing to detection, W).

**Barger (1971) presented a curve for DT' for broadband signatures using $d' = 1.7$, $a = 0.23$, and the value of $A(d')$ as reported by Watters and Moore (1970). He assumed that the width of the broadband signature, W , responsible for detection is always $0.23f$, and thus $FOM = FOM'$.

Several words of caution are in order, before using Figures 7 and 8:

1. The analysis of the signal spectrum must have been done on a sufficiently narrow band basis to detect any lines or narrow band humps of energy. If only 1/3 octave analyses are performed, for example, both types of features can easily be lost with the possibly erroneous conclusion drawn that the detection is due to a full 1/3 octave band.
2. In the case of broadband signatures, W, defined as the width of that portion of the signature which touches the EDC, must be greater than the critical bandwidth (Swets, Green and Tanner, 1962)* if the data of Green (1960a) is to be considered valid.
3. In the case of broadband signatures, Equations (5-12) or (5-13) must be applied, as appropriate, after the graphical manipulation.

The following example will illustrate the graphical method first described by Barger (1971), but modified as described above. Figure 9 shows a hypothetical signature presented in the one-thirtieth octave bands. It is assumed that there is one line at 3.6 kHz, with the rest of the energy being broadband. Figure 10 shows two EDC's, one for lines and one for 2.3% bandwidth broadband. The inputs to Figure 10 are:

1. NL given by Figure 11 for lines, and by the spectrum level of Figure 11 corrected for 2.3% bandwidth for broadband.
2. DI given by Figure 12.

*The ear can be imagined to be like a comb filter consisting of a large number of narrow adjacent filter bands, called the critical bands of hearing. The bandwidth of a critical band is then called its critical bandwidth. While the concept of a critical bandwidth is well accepted, there is disagreement on the numerical values. The most widely used values are about a constant 65 Hz bandwidth for signals up to 1000 Hz, and rising proportionally to the signal frequency above that (500 Hz at 8000 Hz). The range of values determined by various investigators cited in the reference is 40 to 160 Hz for 1000 Hz signals. This indicates that W should be at least three 1/30 octave bands wide above 1000 Hz and more below.

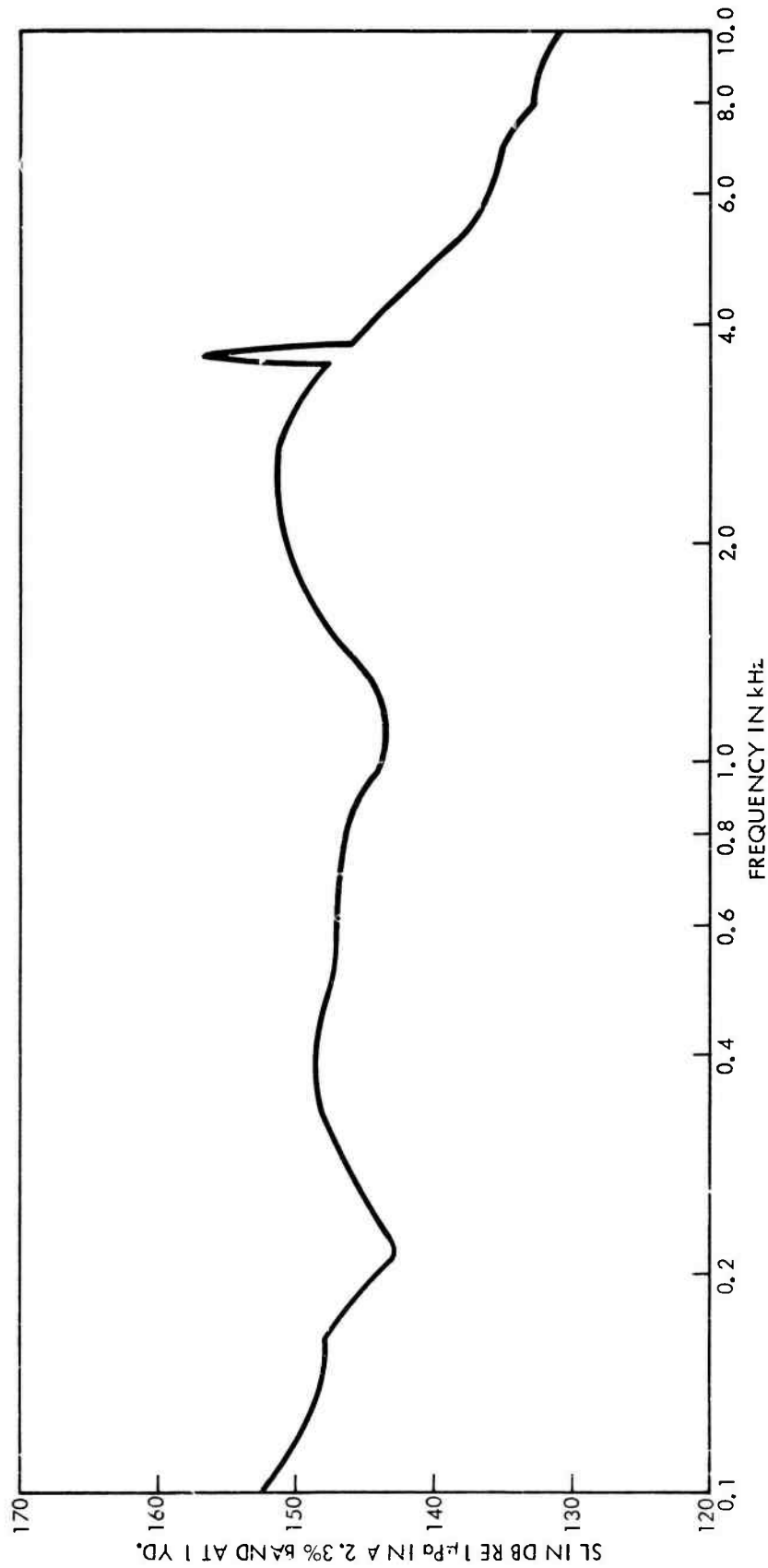


FIG. 9. HYPOTHETICAL TARGET SIGNATURE

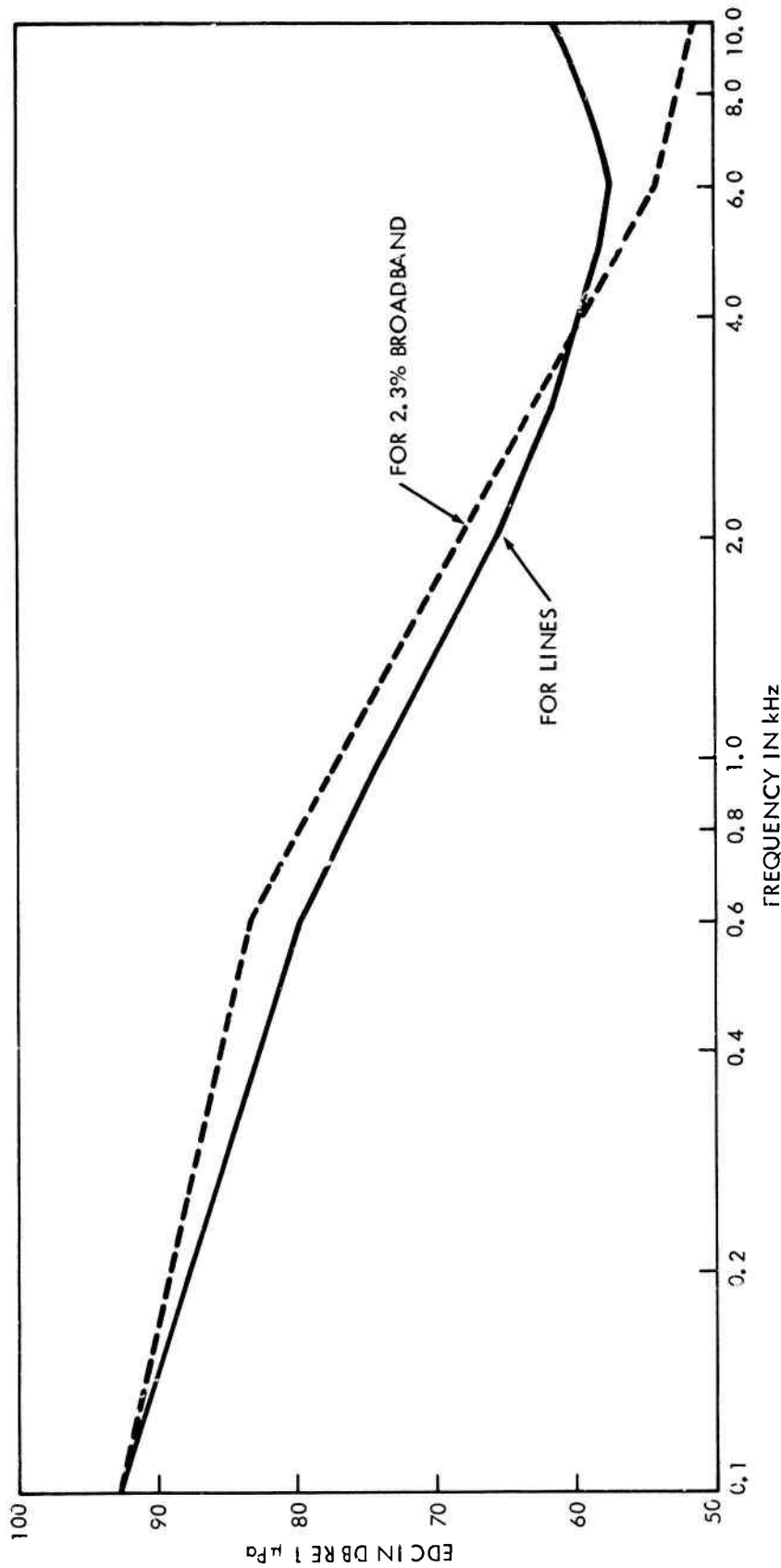


FIG. 10. EQUAL DETECTABILITY CURVES

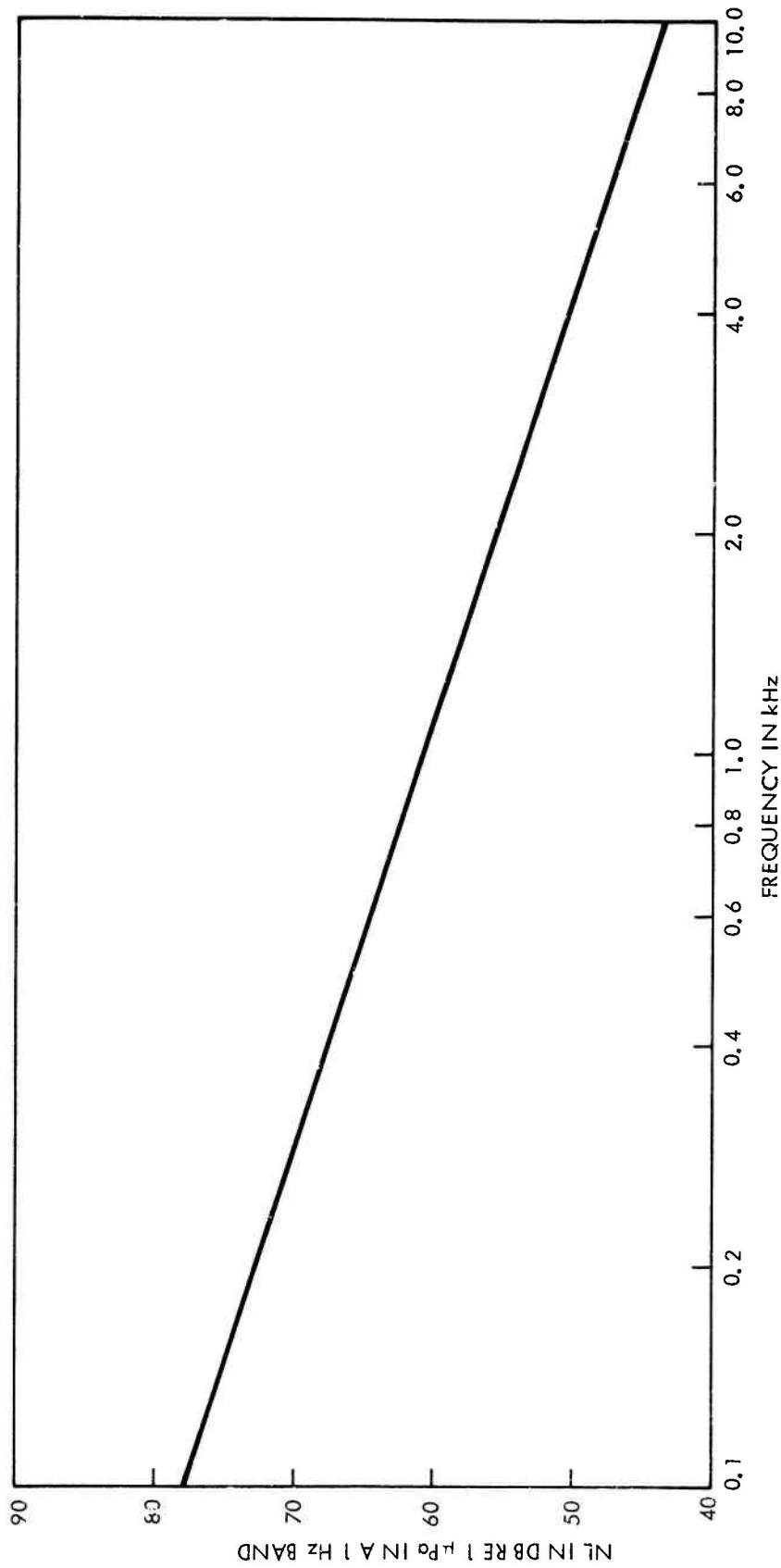


FIG. 11. HYPOTHETICAL NOISE LEVEL

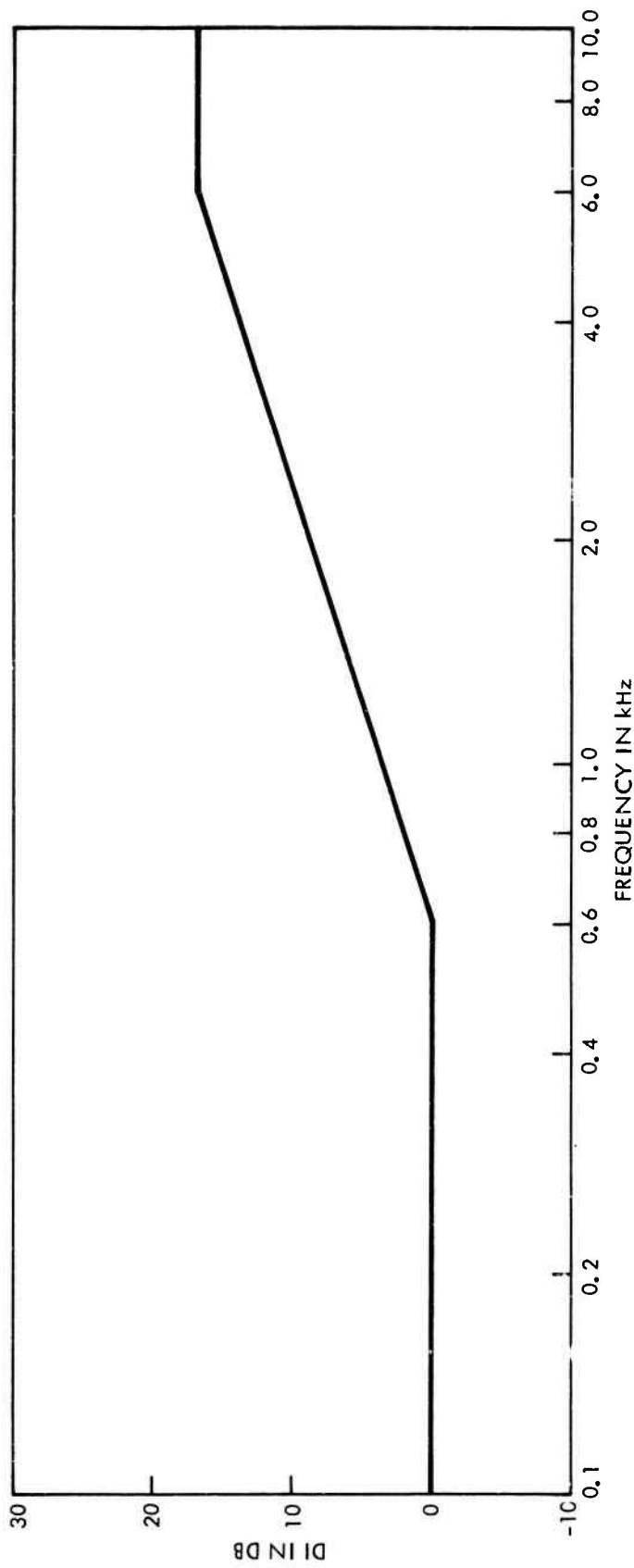


FIG. 12. HYPOTHETICAL DIRECTIVITY INDEX

3. DT and DT' from Figures 7 and 8, respectively, corresponding to $d' = 1.5$ (see Table 4-1 for P(D) and P(FA) which correspond to $d' = 1.5$).
4. $a = 0.023$.

Using the EDC for lines, one sees that the FOM = 96.3 db and the frequency of detection is 3.6 kHz. Using the EDC for broadband noise, one sees that the FOM' = 87.3 db and the frequency of detection is 3 kHz. From Figure 9, the width of the hump at 3 kHz is 2.2 kHz.⁺ Therefore,

$$\begin{aligned} \text{FOM} &= \text{FOM}' + 5 \log \left(\frac{2200}{(0.023)(3000)} \right) & (5-14) \\ &= 87.3 + 7.5 \\ &= 94.8 \text{ db}^* \end{aligned}$$

The procedure given above is adequate for calculating the Figure Of Merit of an uncomplicated signature like the one given as an example. However, when several lines appear in the signature, their presence must be considered. The question of detecting multiple component signals was studied by Green (1958). He derived an expression for the detectability index of lines of different frequency occurring simultaneously. For the case where each line is at least a critical bandwidth removed from every other line

⁺The procedure used here for determining the width of the hump is simple. The edges of the band are taken as the frequencies where the separation between the Equal Detectability Curve and the signature first reaches 3 db. In the example given, this occurs at 2.2 and 4.4 kHz. The justification for this procedure is not rigorous. It is based on an analogy to the normal bandpass filtering problem, which actually passes an amount of energy equal to what would go through a filter with zero loss within the same bandpass, and infinite loss outside the band. This certainly is an area deserving of more study.

^{*}It is interesting to note that the broadband portion of the signature is predicted to be only 1.5 db less detectable (an FOM of 94.8 db as opposed to 96.3 db) than the line which stands up about 9.5 db above the broadband noise. This is due to the width of the broadband signal.

$$d' = \left[\sum_{i=1}^n (d'_i)^2 \right]^{1/2} \quad (5-15)$$

where d' is the index for line i . Green experimentally verified Equation (5-15) for the case of two lines. Green, et al. (1959), later verified this Equation for 16 sinusoids. If the frequencies are separated by less than a critical bandwidth, Green's model for detecting multiple component signals is much more complicated, and apparently has not been tested experimentally. In the case of two lines of equal magnitude, his complicated expression has the correct limits, approaching $\sqrt{2} d'$ in the limit of large separation and approaching $2 d'$ in the limit of zero separation.

The concept of multiple line detection was applied by Barger (1971) for the case where there is more than one coincidence, or near coincidence, of lines with the EDC. Andrews and Hovater (1971) also noted the effects of the presence of multiple lines on detection. The following method is proposed as a general method for treating the problem when the several lines present do not necessarily intersect the EDC together.

Equation (5-15) provides an expression for the overall equivalent d' for all the lines if the d'_i is known for one relationship among the d'_i . The $(n-1)$ other relationships can be defined by assuming

$$A(d'_1) - A(d'_i) = FOM_1 - FOM_i = \delta_{i1} \quad (5-16)$$

where FOM_i is the Figure Of Merit calculated for line i using the EDC (which assumes a particular value of d'_i) for line 1; line 1 being the line of first intersection. The d' curve used is the one corresponding to the frequency of the primary line, since the application of the EDC has accounted for the frequency dependence of d' .

Equation (5-16) positions the different d'_i along a curve, relative to one another. Equation (5-15) fixes their absolute position. The increase in detectability, then, is given by

$$\Delta FOM = A(d') - A(d'_1), \quad (5-17)$$

where, again, the A 's are determined from the same d' curve.

An example of the above procedure will be shown based on the hypothetical signature of Figure 13. The Equal Detectability Curve for lines from Figure 10 is applied, resulting in the individual FOM_i values for each of the three lines as listed in Table 5-1. The second column lists the δ_{i1} values, as defined in Equation (5-16), relative to Line 1. Now we want to have $d' = 1.5$ since the line detectability curve of Figure 10 is based on that value. Therefore, we estimate a $d'_1 = 1.1$ for Line 1 on the $f = 3$ kHz line* of Figure 6, corresponding to $A(d'_1)$ of 19.3 db. From Equation (5-16), $A(d'_2)$ must be 1.0 db less, or 18.3 db. Therefore, d'_2 is 0.83. Similarly, $A(d'_3)$ is 17.0 db and d'_3 is 0.55. This gives a resultant overall d' of 1.48 from Equation (5-15) which is very close to the target value of 1.50. The final values listed in Table 5-1 were obtained by raising each of the values obtained from the initial estimate by 0.02. The value of $A(d')$ corresponding to $d' = 1.5$ is 20.7 db so the ΔFOM from Equation (5-17) is 1.4 db. The final FOM is increased 1.4 db by the two extra lines, over the value of 96.3 db for the highest individual line, for a total of 97.7 db.

	FOM _i	δ_{i1}	d'_i
Line 1	96.3 db	0.0 db	1.12 db
Line 2	95.3 db	1.0 db	0.85 db
Line 3	94.0 db	2.3 db	0.57 db

Table 5-1. Values for Variables in Sample Calculation of Multiple Line Detection.

Two well separated lines, both just touching the EDC, add 1.5 db to the FOM over what either line gives by itself. Similarly, three independent lines all touching the EDC add 2.2 db. Table 5-2 has been constructed to give further guidance on the effect of two well separated lines. It lists the increase in FOM produced by the lower of two lines when its effect is added to the FOM from the higher. Table 5-3 gives additional guidelines for when to stop considering the effect of multiple lines due to negligible increase in FOM (less than 0.1 db). Two extreme situations are covered in the Table. The first is for one line touching the EDC, while

*It makes no difference which frequency line is chosen on Figure 6 since they are all parallel. The frequency dependence is in the EDC of Figure 10.

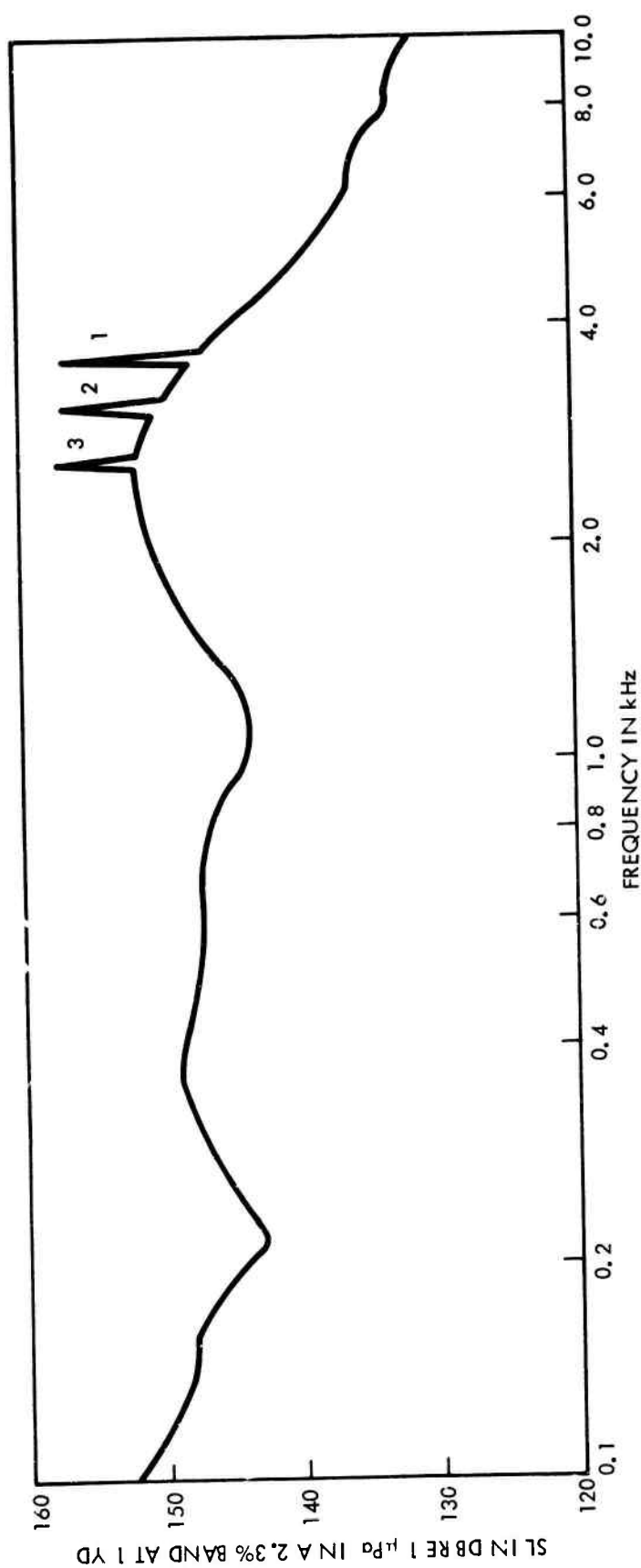


FIG. 13. HYPOTHETICAL SIGNATURE WITH MULTIPLE LINE COMPONENTS

there are anywhere from one to four lines equally far below the EDC. The second situation is for the line in question to lie below the EDC, while there are one to four other lines touching the EDC. In summary, an exact procedure has been given to handle the multiple line situation. In addition, several examples provide guidance as to the magnitude of the increase produced by multiple lines and limits below which their effects are negligible. With these guides, reasonable estimates can be made as to the effect of multiple lines in those cases where it is not necessary to make a more exact calculation.

db Down Relative to Higher Line	Increase in FOM
0 db	1.5 db
1 db	1.1 db
2 db	0.7 db
3 db	0.4 db
4 db	0.2 db
5 db	0.1 db

Table 5-2. Increase in FOM Due to a Second Line as a Function of its Relative Amplitude

NUMBER OF LINES	FOM ₁ - FOM _i	
	ONE LINE TOUCHING EDC (N-1) LINES BELOW	(N-1) LINES TOUCHING EDC, ONE LINE BELOW
2	5.2 db	5.2 db
3	5.9 db	3.8 db
4	6.2 db	3.1 db
5	6.4 db	2.6 db

Table 5-3. Difference in FOM Level Necessary Before the *i*th Line Can be Neglected in Multiple Line Detection

A final refinement to the procedure of predicting detection ranges is the concept of sequential observations. As was discussed earlier, most underwater acoustic sources of interest are continuous ones in comparison to *T*, the integration time.

This affords the sonar operator the luxury of withholding his decision, as to the presence of a target, until after several observations. The operator can, in effect, trade time-until-decision for increased certainty.

Suppose, for example, there is a variable cost for each observation as well as a fixed reward (or fine) for a correct (or incorrect) decision. Swets and Green (1961) showed that the number of observations is an inverse function of the cost per observation. The sonar operator is faced with a similar problem and will follow the same behavior pattern.

One theory (Swets and Green, 1961) says that the detectability index following the n 'th observation is related to the value of d'_1 for the first observation as

$$d'_n = \frac{n(\mu_{SN} - \mu_N)}{n^{1/2}\sigma_N} = n^{1/2} d'_1. \quad (5-18)$$

Equation (5-18) says that the distribution of the sum of n random variables has mean equal to the sum of the means $[n(\mu_N - \mu_N)]$, and variance equal to the sum of the variances $(n\sigma_N^2)$. This result is equivalent to Equation (5-15) when all the d'_i are the same.

Experimental results generally disprove Equation (5-18). Pollack (1959) found that Equation (5-18) consistently overestimated his experimental results where word intelligibility was measured as a function of repeated presentations of a word in noise. Swets and Green (1961) found that an observer's performance in detecting a sinusoidal signal in noise falls below that predicted by Equation (5-18) for $n > 5$. Watters and Moore (1970) found that the ability of their subjects to integrate separate clues was less than that predicted by Equation (5-18), when their task was to detect a line component of a submarine's radiated noise signature when masked by stern aspect radiated noise from a torpedo. In general, one can see from Equation (5-18) that as n gets inordinately large, so does d'_n , which is not reasonable.

Another theory, advanced by Andrews and Hovater (1971), suggests that the effect of sequential observations is binomial,

i.e., that

$$P(D) = \sum_{m=c}^n \frac{n!}{m!(n-m)!} p_D^m (1-p_D)^{n-m} \quad (5-19)$$

and

$$P(FA) = \sum_{m=c}^n \frac{n!}{m!(n-m)!} p_{FA}^m (1-p_{FA})^{n-m} \quad (5-20)$$

where p_D and p_{FA} are the probabilities of detection and false alarm, respectively, for an individual observation, and c is some number of the n observations which exceed the operating level. This theory allows the parameter, c , to be chosen in a fortuitous way. The chief criticisms of this model are (1) a binomial process such as Equation (5-19) and (5-20) assumes perfect memory and (2) Andrews and Hovater do not test the model against experimental data.

It seems reasonable that there will be upper limits to c and n beyond which the human performance will not typify perfect memory. These limits will probably be functions of d' as well. Andrews and Hovater do investigate several hypothetical cases with promising results. But these cases are not tied to the reality of an experiment and use $n = 60$ and $c = 10$ or 20 . One might suspect that human performance would decline before these high values are reached, as the results of Swets and Green (1961) indicate.

While other models (see, for example, Boehme and Weidmann (1970); Iglehart (1966); and Birdsall and Roberts (1965)) have been proposed, it is clear that there is no established theory to describe the effects of sequential observations. It is also clear that the effect must be considered in predicting detection range. Sequential observations, therefore, remain a prime area for further study.

CHAPTER 6

DISCUSSION

Purpose of Study

Before evaluating the accomplishments of this report, it is desirable to understand the original objective of the study and how it evolved into its present form. The original objective was to study the field of psychoacoustics in sufficient depth to judge its accuracy and usefulness in predicting alertment and masking performance of a fleet weapon. It soon became clear that there were differences in predictions among the various works in the field. It also appeared that several seemingly important factors were not being treated. The study progressed then to an evaluation of the importance of these factors and investigations as to how they could be taken into account. Techniques were developed to account for some of the factors, while others appeared to require further research before satisfying answers will be found. The final purpose of the study thus became threefold; to compile a status report on the field of psychacoustics applied to passive sonar, to attempt to advance the field by documenting areas of disagreement with current doctrine and presenting the authors' proposals for handling certain of the additional factors and finally to list those areas which appear to need further study. Many of the questions discussed in Chapter 4 have been studied and reported on individually to some degree in the literature. However, not all of the effects have been applied to the passive sonar problem. What is new here is the attempt to find ways to combine all these effects and determine their application to the detectability index. The end result is a set of curves for detectability index, Figure 6, which is substantially different from those developed or used previously.

Chapter 5 attempts to define the detection threshold for a human working with passive sonar. The choice of an integration time for the ear is reviewed. Combining this with the curves for detectability index of Figure 6 gives the desired end result: curves of detection threshold, Figure 7 for lines, and Figure 8 for broadband noise. Again these are more than trivially different from others in the literature. The concept of an Equal Detectability Curve is reviewed. This is a graphical technique for solving the sonar equation as a function of frequency and determining which feature of

a noise signature will predict first detection or last unmasking. If broadband noise is involved, a technique is presented for utilizing the actual width of the signal being heard, rather than the usual assumption that the bandwidth involved is one third octave. If lines are causing detection, a system is proposed for handling the effect of multiple lines, each of which may be contributing to the detectability. Finally, the effects of sequential observations are discussed, and the conclusion is reached that this question has not yet been brought under control.

The model for the detection threshold of the human ear outlined in Chapter 5 is quite complex in comparison to the simple one cited by Urlick (1967). However, the present model addresses many important questions such as the dependence of the detection threshold on type and frequency of signal. This advantage far outweighs the extra complexity, for it permits focusing attention on those portions of an acoustic signature which are contributing to detection. Thus noise reduction efforts can be concentrated on certain limited frequency bands rather than across the board. It is important from both cost and time standpoints that it be possible to develop mathematical techniques and models for predicting alertment and masking for various weapons from acoustic signatures alone. The alternative of huge numbers of sea trials, or even simulator trials with human operators is simply not practical, excepting as a check on the adequacy of the model.

It should be emphasized that the proposals made here are not considered to be the last word. Instead they are being advanced for consideration and criticism by others. The application of psychoacoustics to passive sonar alertment and masking is still in an early stage of development.

Accomplishments

Chapters 2 and 3 are basically a review of the literature and contain little that is new. Chapter 2 covers the statistical theory of signal detection starting with the fundamental work of Peterson, Birdsall and Fox (1954). Chapter 3 is a review of the literature regarding the basic application of signal detection theory to psychoacoustics. Tanner, et al. (1956), showed rather convincingly that the theory of signal detectability is indeed the proper way to proceed in studying the response of the human ear to aural stimulus. Green, et al. (1959), and Green (1960a) determined just what the relationship is between the detectability of a signal and its energy. It is possible for the reader to secure the basic education needed

in this field from a careful study of this summary. For those desiring to go further in depth into some aspect, an extensive bibliography of original source material is cited.

Chapter 4 investigates some of the differences between the relatively simple laboratory tests reported in the literature and the complex problems of actual passive sonar operation. It starts with the problems of choosing an operating level and having the observer stick to that level. Much of the literature presents results for n Alternative Forced Choice tests. Sonar is more nearly related to the yes-no situation which reduces the detectability. Almost all tests are done by instantaneously gating the signal on and off, whereas a sonar signal as heard by the ear will increase relatively slowly in level, again making it less detectable. Working in the opposite direction, real signals have fluctuations due to the medium which tend to make them more detectable. Uncertainties in exactly what frequency or character the signal will have, and when it will show up in time, both reduce the detectability.

Effect of Errors

It is desirable to have some indication of the effects of any errors or uncertainties in the determination of the detection threshold. An elementary way to get this is to consider the precision of the data which underlies the theory of detection threshold for the ear, and then to translate this into potential uncertainty of range in the ocean for alertment or masking. As an example, consider the data of Green, et al. (1959), for tones. These are based on the results of 25 trials, by each of 11 observers, at 10 signal levels, for 16 different frequencies, or 44,000 observations. This is a rather thorough experiment. Even so, the authors estimate the variance about a given value of $P(c)$ to be

$$\sigma^2 = P(c) [1 - P(c)]/25, \quad (6-1)$$

where 25 is the number of observations at each point. Table 6-1 lists the effect of this uncertainty on the detectability at various values of d' . An inspection of the data of Green (1960a) for noise bursts reveals that similar uncertainties exist there also.

d'	ΔDT due to $\Delta P(c)$
0.5	2.2 db
1.0	1.5 db
1.5	1.6 db
2.0	2.1 db

Table 6-1. Uncertainty in Detection Threshold Due to Uncertainty in $P(c)$.

To translate the db uncertainty given in Table 6-1 into terms more relevant to the problem of the real world, Table 6-2 presents the uncertainty in detection range resulting from an uncertainty of 1.6 db in DT for $d' = 1.5$. Simple spherical spreading has been assumed since any better assumption requires knowledge of local factors such as velocity profile, frequency, water depth, sea state, and bottom conditions. Attenuation and effects of velocity gradients would combine to make the largest values somewhat unrealistic in the real world situation, but the 1 - 10 kyd ranges would certainly be affected that much. Furthermore, Table 6-2 takes only one source of uncertainty into account. There are many more. It is important, therefore, to make these models as accurate as possible.

FOM (db)	Range (kyds)	Spread (kyds)
100	100	83 - 120
90	32	26 - 38
80	10	8.3 - 12
70	3.2	2.6 - 3.8
60	1.0	0.8 - 1.2

Table 6-2. Uncertainty in Detection Range Due to Uncertainty in $P(c)$.

Areas for Further Work

It has already been pointed out that this report does not claim to represent the last word on passive sonar detection. Suggestions have been made as to how to handle a number of problems relating to the application of psychoacoustics to passive sonar. However, there are a number of other areas where the basic experiments and knowledge available seem inadequate.

One very important assumption concerns the way the uncertainties outlined in Chapter 4 add. This assumption, although reasonable, is not founded on any experimental evidence since no one has considered the combined effect of time and frequency uncertainty, for example. Here is one place where further experimental study might resolve an important question.

Much of the discussion outlined in Chapters 3 through 5 is justified by experiments using a 1 kHz tone. While it seems logical that these phenomena can be considered independent of frequency and character of signal, a recent report by Moore (1972) points to an example where the assumption may not be valid. Moore interprets the integration time for a 1 kHz tone, as reported by Licklinder and Green (1961), to be 0.45 sec. He interprets the integration time for broadband signals, as reported by Green (1960a), to be 0.30 sec. This difference in integration time would reduce the Figure Of Merit of a broadband signal by 1 db. As pointed out earlier, we feel that Green's (1960a) data does not permit determination of the break point to this degree of precision. This is a good example of an area where additional thorough experimentation can better determine the dependence of integration time, if any, on the type and frequency of the signal.

It would seem logical that the decrement to performance caused by a slowly increasing signal (Horton, 1959) as well as that due to uncertainty in signal frequency (Green, 1961) would be a function of the detectability index, as were the other corrections. This assumption was made in the derivation of Equation (4-12). Also, as Horton points out, this effect is most pronounced when the character of the signal is like that of the noise. It would seem, then, that broadband signals would be more adversely affected than tonal ones. Here, again, detailed experimentation could answer these questions.

What is the effect of multiple lines when their separation is less than one critical bandwidth? While Green (1958) presented a plausible approach to this question, it has not been tested experimentally.

The effect of multiple bands of noise has not been defined. The effects of simply widening a single band of noise has been covered, but what happens when the two bands are well separated in frequency? Possibly the effects are additive in the same way that two lines separated by more than the critical bandwidth are, but experimental evidence of this is lacking.

Combining the effects of both lines and bands of noise also needs further investigation. This is a very important question since the most detectable line often will lie just a little bit above a broadband of nearly white noise.

The subject of the effect of sequential observations has already been pointed out earlier in the report as a field where only theories exist as to how to handle the problem. These theories do not always agree, and there is little experimental evidence to define the best way to proceed.

Of course, the final judgement of any model of auditory detection depends for validation on a real world test of it. Such a test could be carried out by using samples from the profusion of existing tapes on which are recorded the acoustic signatures of a variety of noise-makers, from torpedoes to submarines. Facilities exist where such tapes can be combined if desired with background noise and the resultant shaped appropriately to simulate the desired NL and DI. Judicious use of notch filters could be employed to eliminate single lines or one of two separated wideband signals suspected of causing detection. Such studies should be able to answer questions such as:

- a. Does the theory of signal detectability accurately predict the Figure Of Merit obtained in a real world experiment?
- b. Is this Figure Of Merit due to lines or broadband signal?
- c. Which lines are causing detection?
- d. What region of the broadband signature is causing detection?
- e. What is the effect on FOM of removing a given line from the signature, i.e., what is the effect of multiple line components?
- f. What are the effects of the various real world uncertainties outlined in Chapter 4 as compared to the controlled laboratory experiments of Green, et al. (1959), and Green (1960a)?
- g. Do broadband and line components in the same signature have the combined effect of increasing the detectability? If so, what is the relationship?

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